Differentiation

Class 1

Example 1 Evaluate $\lim_{x \to 4}(5x+2)$

Example 2 Evaluate $\lim_{x \to 4} \frac{x^2-16}{x-4}$

Example 3 $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$

Example 4 Evaluate $\lim_{x \to 3} \frac{x-3}{\sqrt{x+6}-3}$

Example 5 Evaluate $\lim_{x \to \infty} \frac{5x-1}{3x-4}$

Example 6 Evaluate (*i*) $\lim_{A \to \vartheta} \frac{\sin 3\vartheta}{\vartheta}$ (*ii*) $\lim_{A \to \vartheta} \frac{\sin 5\vartheta}{\tan 4\vartheta}$

Example 7 Evaluate $\lim_{x \to \vartheta} \frac{\cos 5x - \cos 3x}{x^2}$

Example 8 Differentiate from first principals $y = x^2$.

Example 9 Find $\frac{dy}{dx}$ if

(*i*) $y = 4x^2 + 3x + 6$

(*ii*) $y = 6x^4 - 2x^2$

Example 10 Find $\frac{dy}{dx}$ if

$(i)\quad y=\frac{1}{x^2}$

$(ii)\quad y=\sqrt{x}$

$(iii)\ y=\frac{1}{\sqrt{x}}$

Class 2

Example 1 If $y=\frac{3x-5}{7x-2}$ find $\frac{dy}{dx}$

Example 2 For each of the following find $\frac{dy}{dx}$

$(i)\quad y=x\sin x$

$(ii)\quad y=x^2\cos x$

Example 3 If $y=(3x+2)^4$ find $\frac{dy}{dx}$

Example 4 Differentiate each of the following:

(i) $y=(7-3x)^5$

(ii) $y=\sqrt{3x-2}$

(iii) $y=\frac{1}{5x-2}$

(iv) $y=\cos 5x$

(v) $y=\cos^5 x$

Example 5 Differentiate $y=(3x+1)(2x+4)^3$ with respect to x.

Class 3

Example 1 Differentiate $x^2 + xy + y^2 = 9$

Example 2 Differentiate $\sin y = x^2$

Example 3 Differentiate each of the following

(i) $y = \sin^{-1} \frac{x}{3}$

(ii) $y = \tan^{-1} 8x$

(iii) $y = \tan^{-1} \frac{1}{x}$

(iv) $y = x \sin^{-1} x$

Example 4 Find $\frac{dy}{dx}$ if $x = t^2 + 3t$ and $y = 5t + 6$

Example 5 If $x = 4\cos A + 3\sin A$ and $y = 3\cos A - 4\sin A$ find $\frac{dy}{dx}$ when $A = \frac{\pi}{2}$

Example 6 If $y = \sin^3 x$ find $\frac{d^2y}{dx^2}$

Example 7 If $y = kx^2$ where $k \in R$ find the value of k for which

$$x^2 \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = 0$$

Class 4

Example 1 Find $\frac{dy}{dx}$ in each of the following:

(i) $y = e^{3x-4}$

(ii) $y = e^{x^2+3x}$

(iii) $y = x^2 e^x$

(iv) $y = \frac{1}{e^x}$

(v) $y = \frac{e^{x^3}}{e^{9x}}$

Example 2 Differentiate each of the following

(i) $y = \ln(5x-1)$

(ii) $y = \ln\sqrt{4x^2+3x}$

(iii) $y = \ln(3x-5)^6$

(iv) $y = \ln\left(\frac{5x-2}{2x+1}\right)$

(v) $y = \frac{\ln x}{x^2}$

Example 3 Differentiate $y = x^x$

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Class 5

Example 1 Find the equation of the tangent to the curve $y = x^3 - 4x^2 + x + 2$ at the point (1,0)

Example 2 Find tangent to the curve $x^2 + xy + y^2 = 13$ at the point (3,1).

Example 3 Find tangent to the curve given by $x = 3t^2 - 5t + 1$ and $y = 3t - 1$ when $y = 5$.

Example 4 L is a tangent to $y = x^3 - 4x + 1$ at $x = 1$. Find to the nearest degree the measure of the angle formed by the tangent L and the positive sense of the x - axis.

Example 5 Find the co-ordinates of the point on the curve $y = x^2 - 5x + 4$ at which the tangent makes an angle of 135^o with the positive x - axis.

Example 6 Find the points on the curve $y = x^3 + 3x - 2$ where the tangents are parallel to the line $6x - y = 4$.

Example 7 Prove that the curve $y = \frac{2x - 3}{x - 4}$ is decreasing

Example 8 For what values of x is $y = x^2 - 6x - 8$ increasing.

Example 9 Find the values of x for which $y = x^3 - 3x$ decreasing.

Class 6

Example 1 The function $y = 2x^3 + ax^2 + bx$ has turning points when $x = 1$ and $x = -2$. Find the values of a and b.

Example 2 Find the stationary point of $y = xe^{-x}$ and state if this point is maximum or a minimum point.

Example 3 $f(x) = \frac{\ln x}{x}$ where $x > 0$.

(i) Show that the maximum of $f(x)$ occurs at the point $\left(e, \frac{1}{e}\right)$

(ii) Hence, show that $x^e \leq e^x$ for all $x > 0$.

Class 7

Example 1 Determine the number of real roots of the equation $y = x^3 - 3x + 2$.

Example 2 Given that $y = x^3 + kx^2 - 4$ where $x \in R$ and $k > 0$

Find,

(i) the range of values of x for which $f(x) = 0$ has three real roots.

(ii) the range of values of x for which $f(x) = 0$ has three real roots, two of which are equal.

Example 3 Show that $x^3 - 2 = 0$ has a root between 1 and 2. Take $x_1 = 1$ as the first approximate of the real root of this equation, find using the Newton - Raphson method the value of x_2 the second approximation.

Example 4 If $f(x) = x^3 - kx^2 + 8$, $k \in R$. Taking $x_1 = 3$ as the first approximation of one of the roots of $f(x) = 0$, the Newton – Raphson method gives the second approximation as $x_2 = \frac{10}{3}$.

Find the value of k.

Class 8

Example 1 Plot the curve $y = x^3 - 3x^2 + 3$

$$\frac{dy}{dx} = 3x^2 - 6x$$

Example 2 Draw a sketch of the curve $y = \frac{2x+4}{x-3}$ where $x \neq 3$ and $x \in R$.

Example 3 A particle moves along the x - axis; its distance, x meters, from the origin after t seconds is given by $x = t^3 - 3t^2 + 15t - 5$. Find the position and acceleration of the particle when it is at rest.

Example 4 A rocket is fired up in the air. The distance s, in km, reached by the rocket after t seconds is given by $s = 12t - t^2$. Find

(i) the height of the rocket after 2 seconds,

(ii) the time taken to reach the maximum height,

(iii) the maximum height reached.

Example 5 The area of a circle is increasing at a rate of $12m^2 / s$ find the rate of increase of the radius when the radius is $4m$.

Example 6 $x = a(\vartheta + \sin\vartheta)$; $y = a(1 - \cos\vartheta)$ where a is a constant.

Show

$$1 + \left(\frac{dy}{dx}\right)^2 = \sec^2\frac{\vartheta}{2}$$

Proofs Class

Proof 1 Differentiate from first principals $y = x^2$.

Proof 2 Differentiate from first principals $f(x) = x^3$

Proof 3 Differentiate from first principals $f(x) = \frac{1}{x}$

Proof 4 Differentiate from first principals $f(x)=\sqrt{x}$.

Proof 5 Differentiate from first principals $f(x)=\sin x$

Proof 6 Differentiate from first principals $f(x)=\cos x$

Proof 7 To find the $\underset{\vartheta\to\infty}{Lim}\frac{\sin\vartheta}{\vartheta}=\underset{\vartheta\to\infty}{Lim}\frac{\sin\vartheta}{\vartheta}$.

Proof 8 If $y=x^n$ prove $\frac{dy}{dx}=nx^{n-1}$ this is a proof by induction.

Proof 9 To prove the sum rule.

Proof 10 To prove the product rule.

Proof 11 To prove the quotient rule.