Need to Know 2

Differentiation 5

Slope of a tangent

A tangent is a line that hits a curve a one point only.
Since it is a line then we need

(i) a point
(ii) slope

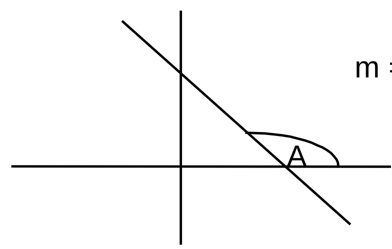
$$m = \frac{dy}{dx} = \text{Slope of tangent at the point } (x, y)$$

Note There are other ways of finding the slope of a curve

(a) Slope between two points $m = \frac{y_2 - y_1}{x_2 - x_1}$

(b) Slope of the line $ax + by + c = 0$ is $m = -{}^{a}/_{b}$.

(c) m $=$ tan of angle with positive sense of x - axis.



m =TanA

Equation of tangent $y - y_1 = m(x - x_1)$

Increasing and Decreasing

$\frac{dy}{dx} > 0$ when we have an increasing slope.

$\frac{dy}{dx} < 0$ when we have a decreasing slope.

Differentiation 6

Maximum and Minimum Points

Stationary points are the same as turning points, which is the same as max or min points.

$$\frac{dy}{dx} = 0$$

Maximum $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$.

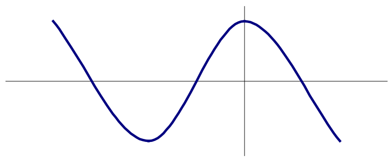
Minimum $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$.

Point of Inflexion $\frac{d^2y}{dx^2} = 0$

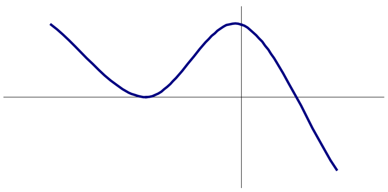
Differentiation 7

To find the number of roots of a cubic equation

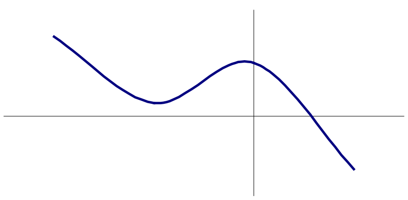
There are four different types of cubic graphs that we may have to make use of



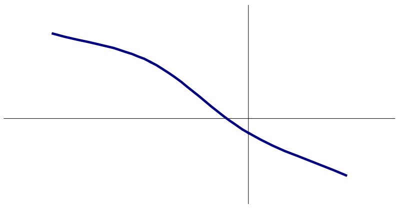
This graph cuts the x - axis in three places.
This cubic has three roots since the max and min points are opposite sides of the x - axis.
One y value is positive and the other negative.



This graph cuts the x - axis in two places.
This cubic has three roots, two of which are equal since the max or min is on the x - axis.
One turning points is on x - axis.
One y value is 0.



This graph cuts the x - axis in one place.
This cubic has one root since the max and min points are on same sides of the x - axis.
Both values of y are positive or both values of y are negative.



This graph cuts the x - axis in one place.
A cubic has one root if there is no max or min point or one of the turning points is also the point of inflection.

The Newton Raphson Formulae

Here we are trying to approximate a root of cubic equation to a required degree of accuracy. To obtain this we follow a method by which a first approximate is used through a process to obtain a second approximate which is then used through the same process to obtain a third approximate and so on, this is called iteration.

If x_n is an approximate to a root of the equation than in general a better approximate is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Differentiation 8

Curve Drawing

Find max and min points
The point of inflexion
Two other points on the curve by substitution

Asymptotes

Horizontal Asymptotes - find the limit as x tends towards infinity (divide above and below by highest power of x). This is the value of y where x does not exist.

Vertical Asymptotes - let the bottom equal zero. This is the value of x for which y does not exist.

Draw a sketch of the curve

Step 1 Find where it cuts x and y axes

Step 2 Differentiate to find if it is increasing or decreasing and to see if there is any stationary point.

Step 3 Find the Asymptotes

Rates of Change

Type 1 Distance, speed, acceleration.

We are given distance (s) as a function of time (t) then

speed or velocity $= \frac{ds}{dt}$ the rate of change of distance with respect to time.

acceleration $= \frac{d^2s}{dt^2}$ the rate of change of speed with respect to time.

Note Is at rest is the same as a velocity of zero which means speed $= 0$

Type 2 When two rates of change are given, either directly or indirectly, and you are asked to find third rate of change related to the other two.

If for example you are given the rate of change of volume with respect to radius, $\frac{dV}{dr}$ and also given the rate of change of radius with respect to time, $\frac{dr}{dt}$ then the rate of change of volume with respect to time is given by

$$\frac{dV}{dt} = \frac{dV}{dr} . \frac{dr}{dt}$$

m / s this is the rate of change of <u>radius</u> with respect to time $= \frac{dr}{dt}$

m^2 / s this is the rate of change of <u>area</u> with respect to time $= \frac{dA}{dt}$

m^3 / s this is the rate of change of <u>volume</u> with respect to time $= \frac{dV}{dt}$

Note Everything changes over time.

To prove identities