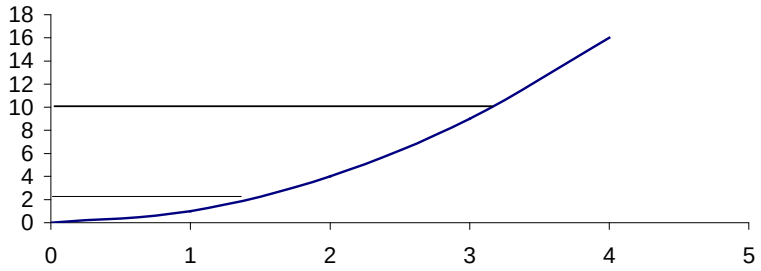
Integration 6

This class has area with the y- axis, the weird and wonderful and volume.

Area with the y- axis

Example 1 Find the area enclosed by the curve $y = x^2$ and the y - axis from $y = 2$ to $y = 10$ where $x > 0$



The area between the curve $x = f(y)$, the y - axis and the lines $y = a$ and $y = b$ is given by

$$\int_a^b f(y)dy = \int_a^b xdy$$

We need to change the equation around to find what $y =$

$$x^2 = y$$

$$x = \sqrt{y}$$

$$x = y^{\frac{1}{2}}$$

$$\text{Area} = \int_2^{10} \left(y^{\frac{1}{2}} \right) dy$$

$$= \frac{2}{3}\left[y^{\frac{3}{2}} \right]_2^{10}$$

$$= \frac{2}{3}\left(\sqrt{10}^3 - \sqrt{2}^3 \right)$$

$$= \frac{2}{3}\left(10\sqrt{10} - 2\sqrt{2} \right)$$

The weird and wonderful

Example 2 Find the area of the bounded region enclosed by the line $y = 2x - 1$, the line $x = 4$ and the curve $y = \frac{1}{x}$, where $x > 0$.

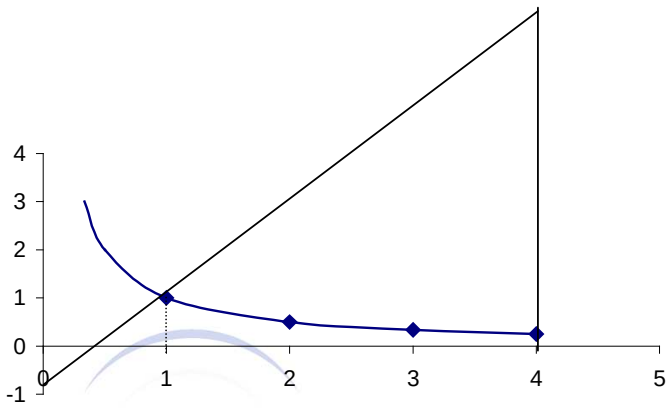
Note In the question they have told us that $x > 0$ which has huge significance, in that we are only go to try to draw the graph for positive values of x.

$y = 2x - 1$ is easy to draw as it is a straight line so we get the points $(0,-1)$ and $\left(\frac{1}{2},0\right)$

$x = 4$ is easy to draw as it is a vertical straight line.

$y = \frac{1}{x}$ is hard to draw as we have no idea as to what it looks like. The idea here is that we put in values of x and find the values of y in order to find points.

x	y
1	1
2	$\frac{1}{2}$
3	$\frac{1}{3}$
4	$\frac{1}{4}$



Must find the point of intersection of $y = \frac{1}{x}$ and $y = 2x - 1$

$$\frac{1}{x} = 2x - 1$$
$$1 = 2x^2 - x$$
$$2x^2 - x - 1 = 0$$
$$(2x+1)(x-1) = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad x = 1$$

$x = 1$ is the value we need since we are told in the question that $x > 0$.

Answer is going to be area under line from 1 to 4 minus area under curve from 1 to 4

Area required $= \int_1^4 (2x-1)dx - \int_1^4 \frac{1}{x}dx = 12 - \ln 4$

Volume of Revolution

The volume generated by rotating the curve $y = f(x)$

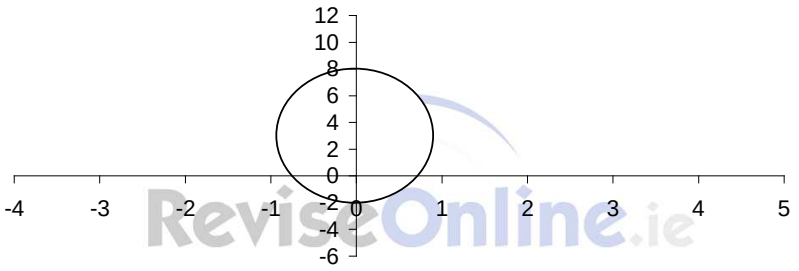
About the x - axis between $x = a$ and $x = b$ is given by $V = \pi \int_a^b y^2 dx$

About the y - axis between $y = a$ and $y = b$ is given by $V = \pi \int_a^b x^2 dy$

Example 3 Find the volume of the sphere generated when the circle $x^2 + (y-3)^2 = 25$ is rotated about the y - axis.

Draw the diagram where the centre is (0,3) and radius is 5.

Since the radius is 5 then y goes from $y = -2$ to $y = 8$



$$x^2 + (y-3)^2 = 25$$

$$x^2 = 25 - (y-3)^2$$

$$x^2 = 25 - (y^2 - 6y + 9)$$

$$x^2 = 25 - y^2 + 6y - 9$$

$$x^2 = 16 + 6y - y^2$$

Volume $V = \pi \int_a^b x^2 dy$

$$V = \pi \int_{-2}^{8} (16 + 6y - y^2) dy$$

$$= \frac{500}{3}\pi \text{ units cubed}$$

Note Volume proofs are in the proof notes.