

Integration Need to Know 1

Integration 1

Basic integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Increase the power by 1 and then divide by the new power.

We must try to write the question in "Standard Form" before we can integrate. This means that we set the question up into 2 different types

Type 1 x^n
 Type 2 Maths tables Page 41

There are 5 basic types of integrals we try with every question.

Type 1 Questions which use the rules of Indices.

Note Once we have found our answer we must put in $+ c$ the constant of integration. If it is forgotten you may lose 3 marks.

Note $x^{\frac{3}{2}} = \left(x^{\frac{1}{2}}\right)^3 = (\sqrt{x})^3 = \sqrt{x}\sqrt{x}\sqrt{x} = x\sqrt{x}$

Type 2 Use factorisation from algebra.

Note Nearly always get the sum or difference of two cubes onto the paper somewhere

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Type 3 Fractions where there is one term on the bottom then split the top.

Type 4 Questions where we need the maths tables on page 41 to write in standard form.

Type 5 Multiply out double brackets or squares.

Integration by Substitution

So far we were dealing with standard integrals as shown in the maths tables. Sometimes we have to change the given question to make it into standard form as follows.

u = bottom (for fractions)

u = part with higher power

u = bit that's annoying

u = bit that when differentiated gives the other

u = guess

Note Must use the rules of logs to tidy up

Integration 2

Definite Integrals

$$\int_a^b f'(x)dx = f(b) - f(a) \text{ where } a \text{ and } b \text{ are called the limits of integration.}$$

Integrate then sub in the top minus sub in the lower limit.

Note Be very careful with signs.

Limits under Substitution

Change limits when we do the substitution.

Don't change the limits and put the values of x in at the end.

Fractions

Type 1 Simple

- (i) Indices.
- (ii) Factorisation.
- (iii) One on bottom the split up the top.
- (iv) Maths tables page 41.

Type 2 Use a substitution. Look back to first class on integration.

Type 3 No factors and no ordinary substitution.

Type 4 By division

Type 5 Completing the square. This is done when we have a quadratic on the bottom only.

To complete the square

Step 1 Insure the coefficient of x^2 is 1.

Step 2 Take half the coefficient of x and square.

Step 3 Write as a perfect square.

Integration 3

Integration of Trigonometric Functions

These can be split up into 7 different types of questions.

Type 1 Maths tables page 9.

Type 2 Substitution to get back to standard form.

Type 3 Integrating Products of Sin and Cos.

Change product to sum or difference using maths tables on page 9.

Note The maths tables has $2 \cos A \sin B = \sin(A + B) - \sin(A - B)$ if there is no 2 in the question we must put $\frac{1}{2}$ in front.

Type 4 Even powers of Sin and Cos.

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Type 5 Sin and cos in the question one of which has an odd power.

When we have Sin and Cos in the same question we must have one to the power of 1 and the other to the power of n. Always let u = the one to the power of n.

Note $-\int_a^b = \int_b^a$ if you change the limits around then you can change the sign in front.

Type 6 Odd powers of Sin and Cos

If we have $\cos^3 x$ then use the fact that

$$\begin{aligned} \cos^3 x &= \cos^2 x \cos x \\ &= (1 - \sin^2 x) \cos x \end{aligned}$$

to create a situation where we have sin and cos in the question.

Type 7 Integrating tan and sec²

Let $u = \tan x$

e^x

From the maths tables $\int e^x dx = e^x + c$

Three different types of questions here.

Type 1 Simple

Two rules that we came across before.

- (i) $\ln e^x = x$
- (ii) $e^{\ln x} = x$

The e and ln cancel each other out.

Note $\log_e x = \ln x$

Type 2 Let $u =$ the power. These will only have one e in them.

Note $e^0 = 1$

Type 3 Let $u = e^x$. These will have more than one e.