Integration Need to Know 2

Integration 4

This class contains special cases and the start of area.

Special Cases

Integrals of the form $\int \sqrt{a^2 - x^2}\,dx$

Let $x = a\sin\vartheta$

Integrals of form

$$\int x^3\sqrt{x^2+1}dx$$

$u = x^2 + 1$

$du = 2xdx$

$\frac{1}{2}du = xdx$

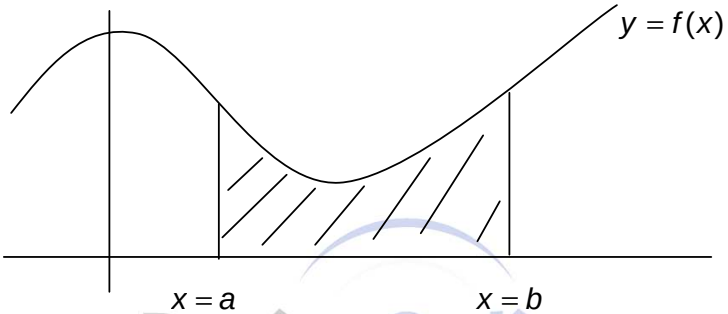
$x^3 = x.x^2$

$u = x^2 + 1 \Rightarrow x^2 = u - 1$

Area under Integration

The area bounded by the curve $y = f(x)$, the x - axis and the lines $x = a$ and $x = b$ is given by

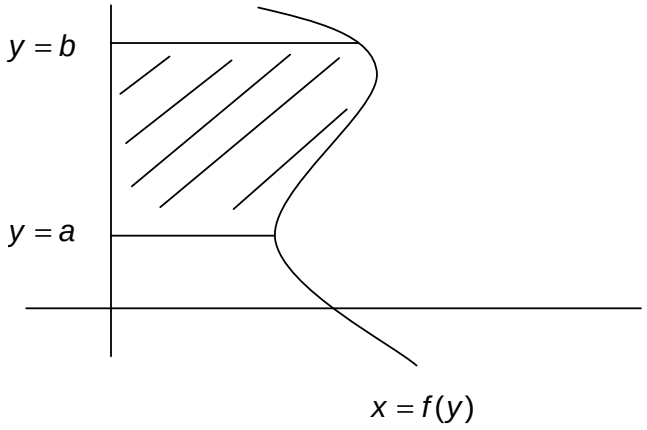
$$\int_a^b f(x)dx = \int_a^b ydx$$



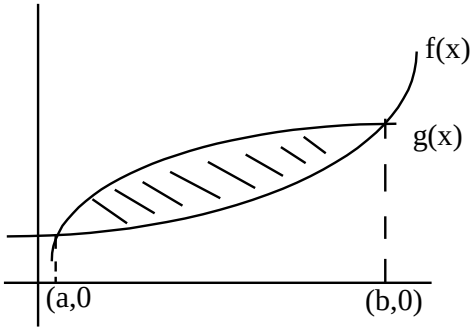
If we require the area between a curve and the y - axis, the function must be written in the form $x = f(y)$

The area between the curve $x = f(y)$, the y - axis and the lines $y = a$ and $y = b$ is given by

$$\int_a^b f(y)dy = \int_a^b xdy$$



Consider the two graphs below where $y = f(x)$ and $y = g(x)$



$$\text{Shaded area} = \int_a^b g(x)dx - \int_a^b f(x)dx$$

Note With any of these questions try to draw a diagram before you start.

Straight lines.

Find where line cuts x - axis so sub in $y = 0$.

Find where line cuts y - axis so sub in $x = 0$.

We need to draw a diagram only to see if the diagram goes below the x axis, because the area below the x axis is negative but the area over the axis is positive.

Integration 5

Quadratic equations

$y = +x^2$ is a u shaped graph.

$y = -x^2$ is a n shaped graph.

In order to draw a quadratic we need to know where it hits the x - axis so we let $y = 0$ and solve the equation.

Cubic equations

2 ways to draw.

(i) Use the factor theorem
(ii) Differentiate and let $= 0$ to find the max and min points

Double Shapes

Draw a rough diagram.

Note The main extra that we have to do when we are asked to find the area between two shapes is to find the point of intersection using simultaneous equations.

Integration 6

Area with the y- axis

The area between the curve $x = f(y)$, the y - axis and the lines $y = a$ and $y = b$ is given by

$$\int_a^b f(y)dy = \int_a^b xdy$$

We need to change the equation around to find what $y =$

$$x^2 = y$$

$$x = \sqrt{y}$$

$$x = y^{\frac{1}{2}}$$

The weird and wonderful

$y = \frac{1}{x}$ is hard to draw as we have no idea as to what it looks like. The idea here is that we put in values of x and find the values of y in order to find points.

Volume of Revolution

The volume generated by rotating the curve $y = f(x)$

About the x - axis between $x = a$ and $x = b$ is given by $V = \pi \int_a^b y^2 dx$

About the y - axis between $y = a$ and $y = b$ is given by $V = \pi \int_a^b x^2 dy$