

Algebra 1

This is by far the most important chapter of the course and a good knowledge of what follows is important for all the other chapters as well as this particular one.

This class will show you the notation you need and how to add, subtract, multiply and long divide.

Notation

$3x^2 + 7x + 9$ is called an expression or a polynomial or a function.

In the $3x^2$ the 3 is called the coefficient - these are the numbers in front of the letters.

The x is called the variable - these are the letters that stand for different numbers.

The 2 is called the power or indice.

In $3x^2 + 7x + 9$ the 9 is called the constant - these are just numbers with no letters attached.

$3x^2 + 7x + 9$ has 3 parts known as 3 terms.

In the number system we must know

N = Natural numbers = whole positive numbers.

Z = Integers = whole positive and negative numbers.

Q = Rational numbers = numbers which can be written as fractions.

R = Real numbers = anything.

To Add and Subtract

Only add like to like - add the numbers in front only not the powers

Note Like terms contain the same letters and have the same powers.

Example 1 Simplify

(i) $2a + 3b - 6a + 3b$

(ii) $2x^2 + 3x - 2x + 7$

(iii) $3x - 1 - (5x - 6)$

$$(i) 2a + 3b - 6a + 3b \\ = -4a + 6b$$

$$(ii) 2x^2 + 3x - 2x + 7 \\ = 2x^2 + x + 7$$

$$(iii) 3x - 1 - (5x - 6) \\ = 3x - 1 - 5x + 6 \\ = -2x + 5$$

To Multiply

When multiplying letters that are the same add the powers

Example 2 Simplify

$$(i) x^2(x^3)$$

$$(ii) (ab)(a^2b^3)$$

$$(i) x^2 \cdot x^3 = x^5$$

$$(ii) (ab)(a^2b^3) = a^3b^4$$

Multiply the number in front and then add the powers on the same letters.

Note If there is no number in front of a letter then the number in front is 1.

Example 3 Simplify

$$(i) 5(6x)$$

$$(ii) 3x(2y)$$

$$(iii) (2x)(3x)$$

$$(iv) 3ab^2(5a^2b^2)$$

$$(i) 5(6x) = 30x$$

$$(ii) 3x(2y) = 6xy$$

$$(iii) (2x)(3x) = 6x^2$$

$$(iv) 3ab^2(5a^2b^2) = 15a^3b^4$$

Dealing with Brackets

Step 1 Multiply everything inside the brackets by what is outside.

Step 2 Add and subtract like terms

Example 4 Simplify each of the following:

$$(i) \quad 2a(b+c) - 3b(a+2c)$$

$$(ii) \quad x(x-1) - 3(x-1)$$

$$(i) \quad 2a(b+c) - 3b(a+2c)$$

$$= 2ab + 2ac - 3ab - 6bc$$

$$= -ab + 2ac - 6bc$$

$$(ii) \quad x(x-1) - 3(x-1)$$

$$= x^2 - x - 3x + 3$$

$$= x^2 - 4x + 3$$

Double Brackets

Split the first up into two and put the second down twice

Example 5 Simplify $(x+3)(x-2)$

$$(x+3)(x-2)$$

$$= x(x-2) + 3(x-2)$$

$$= x^2 - 2x + 3x - 6$$

$$= x^2 + x - 6$$

Example 6 Simplify $(5x-1)(4x-3)$

$$\begin{aligned} & (5x-1)(4x-3) \\ &= 5x(4x-3) - 1(4x-3) \\ &= 20x^2 - 15x - 4x + 3 \\ &= 20x^2 - 19x + 3 \end{aligned}$$

Example 7 Simplify $(2x-3)^2$

Note Square means multiply by itself.

$$\begin{aligned} (2x-3)^2 &= (2x-3)(2x-3) \\ &= 2x(2x-3) - 3(2x-3) \\ &= 4x^2 - 6x - 6x + 9 \\ &= 4x^2 - 12x + 9 \end{aligned}$$

Example 8 Simplify $(4x-3)(x^2-5x-6)$

$$(4x-3)(x^2-5x-6) \quad \text{split the first bracket}$$

$$4x(x^2-5x-6) - 3(x^2-5x-6)$$

$$4x^3 - 20x^2 - 24x - 3x^2 + 15x + 18$$

$$4x^3 - 23x^2 - 9x + 18$$

Long Division

- Step 1 Divide the first into the first and put the answer on top.
- Step 2 Multiply the top by side and put answer underneath
- Step 3 Change the sign of the bottom line and add.

Example 9 Divide $x^3 - 4x^2 + x + 6$ by $x + 1$

Divide x into x^3 and put the answer on top (x^2).

Multiply the top by the side $x^2(x + 1)$ and put the answer underneath

Change the sign of the bottom line and add. Continue the process.

$$\begin{array}{r}
 x^2 - 5x + 6 \\
 x+1 \overline{) x^3 - 4x^2 + x + 6} \\
 \underline{x^3 + x^2} \qquad \qquad \text{change the sign} \\
 -5x^2 + x + 6 \\
 \underline{-5x^2 - 5x} \qquad \qquad \text{change the sign} \\
 6x + 6 \\
 \underline{6x + 6} \\
 0
 \end{array}$$

Example 10 Divide $x^3 - 3x + 2$ by $x - 1$

Note There is no x^2 so when we do the long division we put in $0x^2$.

$$\begin{array}{r}
 x^2 + x - 2 \\
 x-1 \overline{) x^3 + 0x^2 - 3x + 2} \\
 \underline{x^3 - x^2} \\
 x^2 - 3x \\
 \underline{x^2 - x} \\
 -2x + 2 \\
 \underline{-2x + 2} \\
 0
 \end{array}$$