Surds

A surd is a number square rooted

A simple surd is of the form $a\sqrt{b}$. The number in front a is called the rational part and the square root $\sqrt{b}$ is called the irrational part. This is exactly the same as algebra and so most of the rules are the same as the rules in algebra.

Remember if there seems to be no number in front that number is really 1.

Simplifying Surds

Here we are trying to break down surds by getting the number inside the square root as small as possible.

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\sqrt{a^2} = \left(\sqrt{a}\right)^2 = a$$

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

Note What we are looking for is to break up surds in two more surds but one of which we can find the square root of in our heads.

Note Remember what the square roots of 4, 9, 16, 25, 36, 49, 64, 81, 100, 121 and 144 are.

Example 1 Simplify each of the following $(a)\sqrt{75}$ $(b)\sqrt{72}$

(a) $\sqrt{75} = \sqrt{25(3)}$ split 75 ito 25 by 3

$= \sqrt{25}\sqrt{3}$ can split the $\sqrt{25(3)}$ up by rule above

$= 5\sqrt{3}$ since $\sqrt{25} = 5$

(b) $\sqrt{72} = \sqrt{36(2)}$

$= \sqrt{36}\sqrt{2}$

$= 6\sqrt{2}$

Adding and Subtracting Surds

This is the same as in algebra except instead of having letters we have square roots.

Only add like surds. Add the numbers in front.

Example 2 Simplify each of the following:

(a) $\sqrt{2} + \sqrt{3}$ cannot be added as the surds are different

(b) $\sqrt{5} + \sqrt{5} = 2\sqrt{5}$ same as apple + apple = 2 apples

(c) $4\sqrt{3} + 5\sqrt{3} = 9\sqrt{3}$ add the numbers in front

(d) $6\sqrt{7} - 9\sqrt{7} = -3\sqrt{7}$ subtract the numbers in front

Example 3 Simplify each of the following

(a) $\sqrt{8} + \sqrt{32}$
(b) $\sqrt{12} - \sqrt{27}$
(c) $2\sqrt{20} - 3\sqrt{45}$

(a) $\sqrt{8} + \sqrt{32}$

At first glance looks like we cannot add but then we remember from above to try to break each one down and then see what will happen.

$\sqrt{8} = \sqrt{4}\sqrt{2}$

$= 2\sqrt{2}$

Do you see the way that this has $\sqrt{2}$ it is a fair bet the when we look at the other part $\left(\sqrt{32}\right)$ that 2 will go into it and the number we are left with we will be able to square root.

$\sqrt{32} = \sqrt{16}\sqrt{2}$

$= 4\sqrt{2}$

$\sqrt{8} + \sqrt{32}$

$= 2\sqrt{2} + 4\sqrt{2}$

$= 6\sqrt{2}$

(b) $\sqrt{12} - \sqrt{27}$

$\sqrt{12} = \sqrt{4}\sqrt{3}$

$= 2\sqrt{3}$

$\sqrt{27} = \sqrt{9}\sqrt{3}$

$= 3\sqrt{3}$

$\sqrt{12} - \sqrt{27}$

$= 2\sqrt{3} - 3\sqrt{3}$

$= -\sqrt{3}$

(c) $2\sqrt{20} - 3\sqrt{45}$

$\sqrt{20} = \sqrt{4}\sqrt{5}$

$= 2\sqrt{5}$

$\sqrt{45} = \sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$

$2\sqrt{20} - 3\sqrt{45}$

$= 2\left(2\sqrt{5}\right) - 3\left(3\sqrt{5}\right)$

$= 4\sqrt{5} - 9\sqrt{5}$

$= -5\sqrt{5}$

Multiply surds

Done just the same as in algebra so remember the following

$$a(x+y)=ax+ay$$

$$(a+b)(x+y)=ax+ay+bx+by$$

$$\sqrt{a}\sqrt{a}=a$$

Example 4 Simplify out each of the following:

(i) $(2\sqrt{3})(4\sqrt{5})$
(ii) $(4\sqrt{5})(3\sqrt{5})$
(iii) $3(2+\sqrt{5})$
(iv) $2\sqrt{3}(4-5\sqrt{3})$

(i) $(2\sqrt{3})(4\sqrt{5})=8\sqrt{15}$

(ii) $(4\sqrt{5})(3\sqrt{5})=12\sqrt{25}$
$=12(5)=60$

(iii) $3(2+\sqrt{5})=6+3\sqrt{5}$ multiply everything inside by what's outside

(iv) $2\sqrt{3}(4-5\sqrt{3})=8\sqrt{3}-10(3)$
$=8\sqrt{3}-30$

Example 5 Multiply out

(i) $(2+\sqrt{3})(4+\sqrt{3})$

(ii) $(2\sqrt{3}+4\sqrt{7})(2\sqrt{3}-4\sqrt{7})$

(i) $(2+\sqrt{3})(4+\sqrt{3})$ multiply as in algebra

$=2(4+\sqrt{3})+\sqrt{3}(4+\sqrt{3})$

` $=8+2\sqrt{3}+4\sqrt{3}+3$

$=11+6\sqrt{3}$

Example 6 Simplify $\left(\sqrt{12}+\frac{1}{\sqrt{12}}\right)\left(\sqrt{12}-\frac{1}{\sqrt{12}}\right)$

$\left(\sqrt{12}+\frac{1}{\sqrt{12}}\right)\left(\sqrt{12}-\frac{1}{\sqrt{12}}\right)$ difference of 2 squares

$$12-\frac{1}{12}=11\frac{11}{12}$$

If you did not spot the difference of two squares then you could multiply out as normal.

$$\left(\sqrt{12}+\frac{1}{\sqrt{12}}\right)\left(\sqrt{12}-\frac{1}{\sqrt{12}}\right)$$

$$\sqrt{12}\left(\sqrt{12}-\frac{1}{\sqrt{12}}\right)+\frac{1}{\sqrt{12}}\left(\sqrt{12}-\frac{1}{\sqrt{12}}\right)$$

$$12-\frac{\sqrt{12}}{\sqrt{12}}+\frac{\sqrt{12}}{\sqrt{12}}-\frac{1}{12}$$

$$12-\frac{1}{12}=11\frac{11}{12}$$

Division of Surds

When there is a single surd on the bottom multiply above and below by that surd
The surd will not disappear altogether but there will be no surd on the bottom.

Example 7 Simplify $\frac{4}{\sqrt{2}}$ multiply above and below by $\sqrt{2}$

$$\frac{4}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}}=\frac{4\sqrt{2}}{2}=2\sqrt{2}$$

Surd equations

These are question with a square root and an equals sign.

Note Square and square root cancel each other out.

Step 1 Bring the square root to one side and everything else to the other.

Step 2 Square both sides usually to get quadratic equation.

Step 3 Solve the quadratic equation.

Example 8 Solve $x+\sqrt{x}=6$

$\sqrt{x}=6-x$ Isolate the square root

$$\left(\sqrt{x}\right)^2=(6-x)^2$$

$$x=(6-x)(6-x)$$

$$x=6(6-x)-x(6-x)$$

$$x=36-6x-6x+x^2$$

$$x=36-12x+x^2$$

$$x-36+12x-x^2=0$$

$$-x^2+13x-36=0$$

$$x^2-13x+36=0$$

$$(x-4)(x-9)=0$$

$$x=4 \text{ or } x=9$$

Note Must check your answers because when dealing with surd the square root is positive only.

Sub in $x=4$ into $x+\sqrt{x}=6$ to get $4+2=6$ which is a valid answer.

Sub in $x = 9$ into $x + \sqrt{x} = 6$ to get $9 + 3 = 6$ which is an invalid answer.
Only answer to this question is $x = 4$.

Example 9 Simplify

$$\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right) \text{ where } x > 0.$$

Hence solve for x if $\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right) = 8$ where $x > 0$, $x \in R$

$\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right)$ double brackets so split the brackets to multipy out

$$\sqrt{x}\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right) + \frac{3}{\sqrt{x}}\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right)$$

$$x - 3 + 3 - \frac{9}{x}$$

$$x - \frac{9}{x}$$

Replace $\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right)$ with $x - \frac{9}{x}$ since we have worked in out already.

$$\left(\sqrt{x} + \frac{3}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{3}{\sqrt{x}}\right) = 8$$

$x - \frac{9}{x} = 8$ find a common denominator and continue as above

$$x^2 - 9 = 8x$$

$$x^2 - 8x - 9 = 0$$

$$(x - 9)(x + 1) = 0$$

$x - 9 = 0$ or $x + 1 = 0$

$x = 9$ $\quad$ $x = -1$

The only answer is $x = 9$ since we were told in the question $x > 0$.