Indices

There are 6 rules and 3 properties of indices that you must learn before you can do any of this.

Rules

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^0 = 1$$

$$\frac{1}{a^m} = a^{-m}$$

$$\left(a^m\right)^n = a^{mn}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Properties of indices

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a+b)^n \neq a^n + b^n$$

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Evaluations

Example 1 Simplify each of the following

(a) $8^{2/3}$

(b) $64^{-3/2}$

(a) $8^{2/3}$

$$= \left(8^{1/3}\right)^2$$

$$= \left(\sqrt[3]{8}\right)^2$$

$$= 2^2 = 4$$

(b) $64^{-3/2}$

$$= \left(64^{-1/2}\right)^3$$

$$= \left(\frac{1}{64^{1/2}}\right)^3$$

$$= \left(\frac{1}{\sqrt{64}}\right)^3$$

$$= \left(\frac{1}{8}\right)^3 = \frac{1}{512}$$

Note All of the above can be done using a calculator, which is quicker but it is better to use the rules.

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Example 2 Find the value of 2^3 and $27^{-2/3}$ leaving the answer in form $\frac{a}{b}$.

$2^3 = 8$ do not make mistake that $2^3 = 6$

On the calculator put in [8] then use [y^x] button then [3] [=]

$$27^{-2/3} = \frac{1}{27^{2/3}} = \frac{1}{(27^{1/3})^2} = \frac{1}{3^2} = \frac{1}{9}$$

On the calculator put in [27] then use the [y^x] button then [2] then [$a\,^b/_c$] then [3] then [±] [=]

Example 3 Simplify $\frac{3^2 \times 9^{\frac{1}{2}}}{3^4 \times 27^{\frac{2}{3}}}$. Give your answer in the form 3^n, where $n \in Z$

This question can be done using the rules or using the calculator.

Using either method we split up the question. Figure out each part and put back together.

$$9^{\frac{1}{2}} = \sqrt{9} = 3$$

$$27^{2/3} = (27^{1/3})^2 = 3^2$$

$$\frac{3^2 \times 9^{\frac{1}{2}}}{3^4 \times 27^{\frac{2}{3}}} = \frac{3^2 \times 3}{3^4 \times 3^2} = \frac{3^3}{3^6} = 3^{3-6} = 3^{-3}$$

Note $\sqrt{a} = a^{\frac{1}{2}}$ or $a^{\frac{1}{2}} = \sqrt{a}$

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Using the calculator

$$3^2 = 9$$

$$9^{\frac{1}{2}} = \sqrt{9} = 3$$

$3^4 =$ [3] [y^x] 4 = 81

[$27^{\frac{2}{3}}$] = [27] [y^x] [2] [$a\,b/c$] [3] = 9

$$\frac{9\times 3}{81\times 9} = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$$

Unknown in the powers

Example 4 Find the value of x in each of the following

$(a)\ 8^x = \frac{16}{\sqrt{2}}$

$(b)\ 25^x = \frac{1}{125}$

The main idea is to change all the numbers to the same base and then use the rules of indices to have one power = one power so that the powers must be equal.

$8^x = \frac{16}{\sqrt{2}}$ change to the base 2

$(2^3)^x = \frac{2^4}{2^{1/2}}$ use rules of indices $(a^m)^n = a^{mn}$ and $\frac{a^m}{a^n} = a^{m-n}$

$2^{3x} = 2^{4-1/2}$ tidy up

$2^{3x} = 2^{3\,1/2}$ drop the base

$$3x = \frac{7}{2}$$

$$x = \frac{7}{6}$$

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$25^x = \frac{1}{125}$ change to base 5

$(5^2)^x = \frac{1}{5^3}$ use rules of indices $(a^m)^n = a^{mn}$ and $\frac{1}{a^m} = a^{-m}$

$5^{2x} = 5^{-3}$ drop the bases

$2x = -3$ divide across by 2

$x = -\frac{3}{2}$

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