

Algebra 9

This class deals with a difficult section of worded questions.

Worted Questions

Some of the questions will be written in English and you have to make mathematical equations before finding the answers.

Let the unknown number be x

When given a worded question

- (i) Read the question.
- (ii) Read the question and try to write an equation (or equations).
- (iii) Read the question and the equation you have written to see if it makes sense.
- (iv) Solve the equation you have written (whether it is right or wrong).

You must try to write out a dictionary for yourself as you come across different words and expressions. Here are some common terms

Difference means minus

Sum means add

One-third means divide by 3

Six times means multiply by 6.

Is means equals

A year ago is $x - 1$

There are 4 different types of questions in this section.

Simple equations

Example 1 When 3 is added to twice a number the result is 15 find the number.

Let the unknown number be x

$$2x + 3 = 15$$

$$2x = 12$$

$$x = 6$$

Example 2 When 4 is subtracted from three times a number the result is the same as adding 5 to twice the number. Form an equation and hence find the unknown number.

$$3x - 4 = 2x + 5$$

$$3x - 2x = 5 + 4$$

$$x = 9$$

Related objects.

Example 3 John is five times older than his son. In 6 years time he will be three times older than his son. Form an equation to find John's age now.

Age now John's son = x John = $5x$

Age in 5 years John's son = $x + 6$ John = $5x + 6$

$$5x + 6 = 3(x + 6)$$

$$5x + 6 = 3x + 18$$

$$2x = 12$$

$$x = 6$$

John is 30 years old.

Equations that become quadratic.

Example 4 When a number is added to its square the result is 56, find the number.

We let the unknown number be x . Its square is x^2 we add to come up with the equation

$$x + x^2 = 56$$

$$x^2 + x - 56 = 0$$

$$x^2 + 8x - 7x - 56 = 0$$

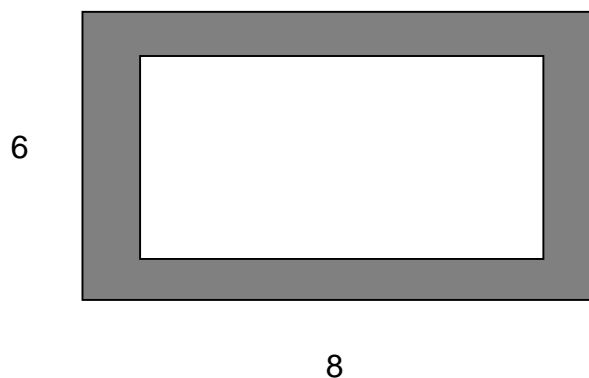
$$x(x+8) - 7(x+8) = 0$$

$$(x-7)(x+8) = 0$$

$$x-7 = 0 \quad \text{or} \quad x+8 = 0$$

$$x = 7 \quad \text{or} \quad x = -8$$

Example 5 The diagram below show a rectangular garden surrounded by a path, which is x wide all the way round. If the area of the path is 13 find the value of x .



Area of the path is the area of the large rectangle minus the area of the small rectangle.

$$\text{Area of a rectangle} = l \times b$$

$$\text{Area of big rectangle} = 6 \times 8 = 48$$

$$\text{Area of small rectangle} = (6 - 2x)(8 - 2x)$$

Area of path = big rectangle – small rectangle.

$$13 = 48 - (6 - 2x)(8 - 2x)$$

Need to multiply out $(6 - 2x)(8 - 2x)$ first.

$$(6 - 2x)(8 - 2x)$$

$$6(8 - 2x) - 2x(8 - 2x)$$

$$48 - 12x - 16x + 4x^2$$

$$48 - 28x + 4x^2$$

$$13 = 48 - (6 - 2x)(8 - 2x)$$

$$13 = 48 - (48 - 28x + 4x^2)$$

$$13 = 48 - 48 + 28x - 4x^2$$

$$13 = 28x - 4x^2$$

$$4x^2 - 28x + 13 = 0$$

$$4x^2 - 2x - 26x + 13 = 0$$

$$2x(x - 1) - 13(2x - 1) = 0$$

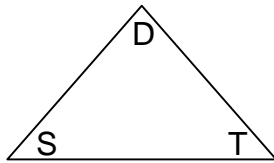
$$(2x - 13)(2x - 1) = 0$$

$$2x - 13 = 0 \quad \text{or} \quad 2x - 1 = 0$$

$$x = \frac{13}{2} \quad \text{or} \quad x = \frac{1}{2}$$

Only possible answer is $x = \frac{1}{2}$

Example 6 A man drove a distance of 100km at x km/h. On the return journey he reduced his speed by 5km/h. If it took him 1 hour longer to get home form an equation and find the value of x .



This is a speed distance time question where we use Dad's Silly Triangle.

Outward.	Distance = 100	Speed = x	Time = $\frac{100}{x}$
Homeward.	Distance = 100	Speed = $x - 5$	Time = $\frac{100}{x - 5}$

Since his speed was slower on the home journey it took him longer (by 1 hour)

$$\frac{100}{x-5} - \frac{100}{x} = 1$$

$$\frac{100x - 100(x-5) = (x-5)x}{(x-5)x}$$

$$100x - 100x + 500 = x^2 - 5x$$

$$500 = x^2 - 5x$$

$$500 - x^2 + 5x = 0$$

$$x^2 - 5x - 500 = 0$$

$$x^2 - 25x + 20x - 500 = 0$$

$$x(x-25) + 20(x-25) = 0$$

$$(x-25)(x+20) = 0$$

$$x - 25 = 0 \quad \text{or} \quad x + 20 = 0$$

$$x = 25 \quad \text{or} \quad x = -20$$

The answer must be $x = 25$ as speed must be positive.

Example 7 €5,000 was divided among a group of people in equal amounts. If there had been 15 fewer people each would have received €75 more. Let x represent the number of winners, write an equation in terms of x from the above information and hence find the number of people.

Let x = the number of people.

$\frac{5000}{x}$ is what each person gets.

$\frac{5000}{x - 15}$ is what each person gets if there were 15 fewer people (each person gets more money)

$$\frac{5000}{x - 15} - \frac{5000}{x} = \frac{75}{1} \quad \text{Bigger amount} - \text{smaller amount} = 75$$

Get the common denominator and solve the quadratic to find $x = 40$

Equations that become simultaneous

Example 8 The sum of two numbers is 7. If 3 times the smaller is taken from 5 times the larger the result is 11. Find the two numbers.

Let one number be x and the other be y and write down clearly which is which.

Let the larger number be x and the smaller number be y .

The sum of two numbers is 7 means $x + y = 7$

3 times the smaller means $3y$

5 times the larger means $5x$

If 3 times the smaller is taken from 5 times the larger the result is 11 means $5x - 3y = 11$

Simultaneous Equations are

$$x + y = 7$$

$$5x - 3y = 11$$

Solve to find $x = 4$ and $y = 3$

Example 9 A bag contains 36 coins, which are either 20c coins or 50c coins. If there are 36 coins in the bag with a total value of €8 find the number of 20c coins and the number of 50c coins

Let the number of 20c coins be x and the number of 50c coins be y .

$$x + y = 36 \text{ as there are 36 coins.}$$

The 20c coins are worth $20x$. The 50c coins are worth $50y$. The total value of the bag of coins is 800

So $20x + 50y = 800$ simplify to

$$2x + 5y = 80$$

Simultaneous Equations are

$$x + y = 36$$

$$2x + 5y = 80$$

Solve to find $x = 20$ and $y = 16$