Algebra 8

This is the eight and last class, which deals with logs only.

Rules of Logs

Definition: $\log_b n = p \Rightarrow b^p = n$

If we know two of the three unknown b, n or p we can find the third one.
Known as the bnp, base ,b, to power of p equals number n.

Example 1 Evaluate $\log_4 64$

$$\log_4 64 = x \quad \text{use defination to change logs to indices}$$
$$4^x = 64 \quad \text{change to same base}$$
$$4^x = 4^3$$
$$x = 3$$

Example 2 Solve for x when $\log_2 x = 5$

$$\log_2 x = 5$$
$$2^5 = x$$
$$x = 32$$

Note If asked to solve an equation with logs we must have the same base

$$\log a + \log b = \log ab$$

$$\log a - \log b = \log \frac{a}{b}$$

$$\log a^n = n \log a$$

$$\log_b a = \frac{\log_x a}{\log_x b}$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

Note We need to know rules backwards.

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$n \log a = \log a^n$$

Watch out for simple mistakes that students who don't really know the rules will do

$$(\log a)^n \neq n \log a$$

$$\frac{\log a}{\log b} \neq \log(a - b)$$

Type 1 Logs equal a constant

Example 3 Solve for x when $\log_2(x+1) + \log_2(x-1) = 3$

Since we have the same base we must follow the method

Step 1 logs to one side numbers to the other.

Step 2 Use rules of logs to have one log = one number.

Step 3 Use definition of logs to get rid of logs and solve.

$\log_2(x+1) + \log_2(x-1) = 3$ add becomes multiply

$\log_2(x+1)(x-1) = 3$ use defination to change logs to indices

$x^2 - 1 = 2^3$ solve the quadratic equation

$$x^2 - 1 = 8$$

$$x^2 = 9$$

$$x = \pm 3$$

Only answer is $x = 3$

Note Must check answer because not allowed log of a negative.

Type 2 Logs on both sides.

Example 4 Solve $\log_2(x+1) - \log_2(x-1) = \log_2 3$

Since we have the same base we must follow the method

Step 1 Use rules of logs to have one log = one log.

Step 2 Drop the logs on both sides and solve.

$$\log_2\left(\frac{x+1}{x-1}\right) = \log_2 3 \qquad \log a - \log b = \log\frac{a}{b}$$

$$\frac{x+1}{x-1} = 3$$ logs on both sides so drop logs and solve the equation

$$x+1 = 3x-3$$

$$x = 2$$

Type 3 Change of base

Example 5 Solve $\log_2 x - \log_4(x-1) = 1$

Note Must change the base and a good rule of thumb that we will use in other parts of the course is **break the bigger one down.**

Must change from base 4 to base 2 by using change of base rule.

$$\log_4(x-1) = \frac{\log_2(x-1)}{\log_2 4}$$

Note Can evaluate the bottom since we have two parts of the definition of logs.

$$\log_2 4 = y$$

$$2^y = 4$$

$$y = 2$$

$$\log_4(x-1) = \frac{\log_2(x-1)}{\log_2 4} = \frac{\log_2(x-1)}{2}$$ replace into question

$$\log_2 x - \log_4(x-1) = 1$$

$$\log_2 x - \frac{\log_2(x-1)}{2} = 1$$

$$2\log_2 x - \log_2(x-1) = 2$$ multiply every term by 2

$$\log_2 x^2 - \log_2(x-1) = 2$$ use rules of logs to solve equation

$$\log_2 \frac{x^2}{x-1} = 2$$

$$\frac{x^2}{x-1} = 2^2$$

$$\frac{x^2}{x-1} = 4$$

$$x^2 = 4x - 4$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$x = 2$$

Example 6 Solve for x if $\log_3 x - 2\log_x 3 = 1$

Note Must get same base on logs, it does not matter which base we change.
Change to the base x.

$\log_3 x = \frac{\log_x x}{\log_x 3} = \frac{1}{\log_x 3}$ replace into the question

$\log_3 x - 2\log_x 3 = 1$

$\frac{1}{\log_x 3} - 2\log_x 3 = 1$ multiply across by $\log_x 3$

$1 - 2(\log_x 3)^2 = \log_x 3$ forms into a quadratic where we let $t = \log_x 3$

$2t^2 + t - 1 = 0$ solve the quadratic to find 2 values of t

$(2t-1)(t+1) = 0$

$t = \frac{1}{2}$ or $t = -1$ use these 2 values of t and definition of logs to find x

$\log_x 3 = \frac{1}{2}$ $\log_x 3 = -1$

$x^{\frac{1}{2}} = 3$ $x^{-1} = 3$

$x = 9$ $\frac{1}{x} = 3$

$x = \frac{1}{3}$

Type 4 x in power then log both sides

When there is a variable in the power and we cannot use indices then we bring logs in on both sides.

This comes in as the last 5 marks in questions.

Example 7 Solve $2^x = 5$

Note Cannot change 5 into 2 to the power of any number since $2^2 = 4$ and $2^3 = 8$ so bring logs in on both sides.

$2^x = 5$ bring in logs on both sides

$\log 2^x = \log 5$ use rules of logs to bring x down

$x \log 2 = \log 5$ divide across by $\log 2$ since it is just a number

$$x = \frac{\log 5}{\log 2}$$