

Algebra

Class 1

Example 1 Simplify $(2x - 3)^2$

Example 2 Factorise each of the following:

(i) $x^2 - 8x$

(ii) $ab + a - b - 1$

(iii) $ac - 3bd - bc + 3ad$

Example 3 Factorise

(i) $2x^2 - 7x + 6$

(ii) $3x^2 - xy - 4y^2$

Example 4 Factorise each of the following.

(i) $4x^2 - 25$

(ii) $36 - (a - b)^2$

Example 5 Factorise $8x^3 + 125y^3$

Example 6 Factorise

(i) $2x^3 - 50x$

(ii) $x^4 - y^4$

Example 7 Solve $x^2 + 5x - 6 = 0$ and hence or otherwise solve

$$(2t - 1)^2 + 5(2t - 1) - 6 = 0$$

Example 8 For what values of $k \in R$ will the equation $x^2 + kx + k - 1 = 0$ have equal roots?

Class 2

Example 1 Show that $\frac{x}{x-3} + \frac{3}{3-x}$ simplifies to a constant where $x \neq 3$.

Example 2 Write as a single fraction $\frac{4}{x-5} - \frac{3}{x^2-25}$ where $x \neq 5$.

Example 3 Solve $\frac{x-1}{x} - \frac{3x}{x-1} = 2$, $x \neq 0$ and $x \neq 1$ where $x \in R$

Example 4 Express as a single fraction $\frac{p - \frac{q^2}{p}}{1 + \frac{q}{p}}$

Example 5 Solve for x and y
 $x + y = 3$
 $x^2 + y^2 = 5$

Example 6 Find the points of intersection of

$$\frac{4}{x} - \frac{3}{y} = 4$$

$$\frac{5}{x} - \frac{6}{y} = 2$$

Example 7 Solve the equations

$$3x + 5y - z = -3 \quad \text{---(i)}$$

$$2x + y - 3z = -9 \quad \text{---(ii)}$$

$$x + 3y + 2z = 7 \quad \text{---(iii)}$$

Example 8 If $7x + 8 = a(x-1) + b(2x+3)$, for all x , find the values of a and b .

Class 3

Example 1 Form the quadratic equation with roots $3 \pm \sqrt{5}$

Example 2 If α and β are the roots of $4x^2 - 5x + 3 = 0$ without solving the equation find the values of

(i) $\alpha^2 + \beta^2$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

Example 3 For what value of k will one of the roots of $x^2 + (3k + 4)x + 3k + 4 = 0$ be three times the other root where $k \in R$.

Example 4 If one root is double the other in the equation $ax^2 + bx + c = 0$ show that $2b^2 = 9ac$

Example 5 If α and β are the roots of $2x^2 - 5x + 13 = 0$ construct a quadratic equation with roots $2\alpha - 1$ and $2\beta - 1$

Example 6 If $\alpha + 3$ and $\beta + 3$ are the roots of $x^2 - 8x + 11 = 0$ form a quadratic with roots α^2 and β^2 .

Class 4

Example 1 Solve $|x + 4| = 2$

Example 2 Solve $|2x - 1| = |x - 3|$

Example 3 Solve the inequality $1 - 2x \geq -5, x \in N$

Example 4 Solve $5 + x < 3x - 1 < 13 + 2x$

Example 5 Find the range of values of x for which $x^2 + 5x + 6 < 0$

Example 6 Find all the real numbers such that $\frac{x+2}{x+1} \leq 2$ where $x \neq -1, x \in R$

Example 7 Solve the following $|1 - 5x| < 9$

Example 8 Prove that

$$(i) \quad a^2 + b^2 \geq 2ab$$

$$(ii) \quad \frac{a}{b} + \frac{b}{a} \geq 2 \quad \text{if } ab > 0$$

Example 9 Prove $a^2 - 8a + 17 > 0$

Example 10 Prove $a^2 + 2ab + 3b^2 \geq 0$.

Example 11 Show that the equation $x^2 + (p + q)x + pq = 0$ has real roots, for all values of p and q where $p \in R$ and $q \in R$.

Example 12 If $a > b > c > 0$ prove that $(a - b)(b - c) < 0$

Class 5

Example 1 Given that $h(x) = x^2$, write down $h(x+3)$.
Hence, find the values of x for which $h(x) = h(x+3)$.

Example 2 Let $f(x) = \frac{1}{x}$ for all $x \in R$ and $x \neq 0$

Points a and b have coordinates $(p, f(p))$ and $(q, f(q))$, respectively, for $0 < p < q$

(i) Show that the equation of the line ab can be written as

$$y = g(x) = \frac{1}{p} - \frac{1}{pq}(x - p).$$

(ii) Show that

$$f(x) - g(x) = \frac{(x - q)(x - p)}{pqx}.$$

Hence, show that $f(x) - g(x) < 0$ for $0 < p < x < q$

Example 3 Factorise $2x^3 - 5x^2 + x + 2 = 0$

Example 4 If $x + 2$ is a factor of $x^3 - 3x + k = 0$ find the value of k and the roots of the cubic.

Example 5 If $f(x) = 2x^3 + ax^2 + bx + 2$ where a and b are real numbers.
Given that $x - 1$ and $x - 2$ are factors of $f(x)$ find the values of a and b .

Example 6 $(x - t)^2$ is a factor of $x^3 + 3px + c$
Show that

(i) $p = -t^2$

(ii) $c = 2t^3$

Class 6

Example 1 Simplify $\sqrt{27}$ and $\sqrt{20}$

Example 2 Simplify $\sqrt{12} + \sqrt{300}$

Example 3 Simplify each of the following:

(i) $3(2 + \sqrt{5})$

(ii) $(2 + \sqrt{3})(4 + \sqrt{3})$

(iii) $\left(\sqrt{12} + \frac{1}{\sqrt{12}}\right)\left(\sqrt{12} - \frac{1}{\sqrt{12}}\right)$

(iv) $\left(x + \sqrt{2} + \frac{1}{x}\right)\left(x - \sqrt{2} + \frac{1}{x}\right)$

Example 4 Simplify $(2 + \sqrt{3})(2 - \sqrt{3})$

Example 5 Simplify $\frac{4}{\sqrt{2}}$

Example 6 Simplify $\frac{3 + \sqrt{5}}{3 - \sqrt{5}}$

Example 7 Simplify $\frac{1}{x + \sqrt{x^2 - 1}}$

Example 8 Solve $x + \sqrt{x} = 6$

Class 7

Example 1 Find the value of 2^3 and $27^{-\frac{2}{3}}$ leaving the answer in form $\frac{a}{b}$ where $a \in R$ and $b \in R$

Example 2 Find the value of x in each of the following:

$$(a) 8^x = \frac{16}{\sqrt{2}}$$

$$(b) 25^x = \frac{1}{125}$$

Example 3 Solve for x if $3^{2x+1} + 26(3^x) - 9 = 0$

Example 4 If for all integers n , $u_n = 3^n + 2^n$ show that $u_{n+2} - 5u_{n+1} + 6u_n = 0$

Class 8

Example 1 Evaluate $\log_4 64$

Example 2 Solve for x when $\log_2 x = 5$

Example 3 Solve for x when $\log_2(x+1) + \log_2(x-1) = 3$

Example 4 Solve $\log_2(x+1) - \log_2(x-1) = \log_2 3$

Example 5 Solve $\log_2 x - \log_4(x-1) = 1$

Example 6 Solve for x if $\log_3 x - 2\log_x 3 = 1$

Example 7 Solve $2^x = 5$

Proofs Class

Proof 1 If one root is three times the other in the equation $ax^2 + bx + c = 0$ show that $3b^2 = 16ac$

Proof 2 If one root is n the other in the equation $ax^2 + bx + c = 0$ show that $n^2b = (n+1)^2ac$

Proof 3 If k is a real number such that $f(k) = 0$, prove that $x - k$ is a factor of $f(x) = ax^3 + bx^2 + cx + d$ where $a, b, c, d \in R$. (This is the factor Theorem)