

Algebra Need To Know 2

Algebra 5

Functions

Note $f(x) = g(x) = h(x)$ all are the same just different letters.

Note They can ask any part of the course in any question.

Note Do the parts you can do and make any attempt at the parts you cannot do.

The Factor Theorem

If $f(a) = 0$, then the remainder = 0 so $x-a$ is a factor of $f(x)$.

This comes down to either one of two methods:

- (i) Sub in or
- (ii) Divide in.

Type 1 Need to know the proof.

Type 2 Given a cubic and asked to factorise.

Type 3 One unknown variable in cubic.

Note Once we have found an unknown we can now use the value of that unknown in the rest of the question.

Note When we are given a cubic equation that has only three terms make it into four by putting in $0x^2$.

Type 4 Two unknown variables in cubic.

Note Because there are two unknowns you should realize that we need a second equations so that we

Type 5 Only variables in both factor and cubic.

Algebra 6

Surds

Simple surd has just one term like $a\sqrt{b}$. a is called the rational part and b the irrational part.

A compound surd has two parts like $a + \sqrt{b}$.

Basic rules that must be remembered at all times

$$(i) \quad \sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$(ii) \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$(iii) \quad \sqrt{a^2} = (\sqrt{a})^2 = a$$

$$(iv) \quad \sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

If asked to simplify a surd factorise the given number into two factors one of which is a perfect square.

Adding surds

Only can add if they have the same irrational part. Add and subtract the numbers in front

Multiply surds

Done just the same as in algebra so remember the following

$$a(x+y) = ax + ay$$

$$(a+b)(x+y) = ax + ay + bx + by$$

$$\sqrt{a}\sqrt{a} = a$$

Surd conjugate

This means that we change the sign in the middle of a compound surd

Compound Surd	Surd Conjugate
$a + \sqrt{b}$	$a - \sqrt{b}$
$\sqrt{a} + \sqrt{b}$	$\sqrt{a} - \sqrt{b}$

When we multiply a compound surd by its conjugate we always end up with a rational number (no surds) because it is the difference of two squares backwards.

Simple fractions

When there is a single surd on the bottom multiply above and below by that surd

The surd will not disappear altogether but there will be no surd on the bottom.

Fractions with Compound Surd

Multiply above and below by the conjugate of the bottom

Surd Equations

Step 1 Bring the square root to one side and everything else to the other. This is vital.

Step 2 Square both sides usually to get quadratic equation.

Step 3 Solve the quadratic equation.

Note Must check your answers because when dealing with surd the square root is positive only.

Algebra 7

Indices

The first thing to do here is to learn off the rules of indices.

$$a^m \times a^n = a^{m+n}$$

$$a^{m+n} = a^m \times a^n$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{m-n} = \frac{a^m}{a^n}$$

$$a^0 = 1$$

$$1 = a^0$$

backwards the rules are

$$\frac{1}{a^m} = a^{-m}$$

$$a^{-m} = \frac{1}{a^m}$$

$$(a^m)^n = a^{mn}$$

$$a^{mn} = (a^m)^n$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Properties of indices

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$(a+b)^n \neq a^n + b^n$$

Type 1 Evaluations.

Type 2 Using the rules to solve for x . We must have everything to the same base.

Type 3 Indices that become quadratics with substitution.

Note Must check answers because we are not allowed to go from a positive base to a negative answer.

Type 4 Questions where we have to expand and prove an equation is true.

Note Split up each part simplify as far as possible and then put back into the question.

Note An easy way to finish this is to let $a = 3^n$ and $b = 2^n$

Note One of the easiest and biggest mistakes to make here is $5 \cdot 3^n = 15^n$ (not true)

Algebra 8

Rules of Logs

Definition: $\log_b n = p \Rightarrow b^p = n$

If we know two of the three unknown b , n or p we can find the third one.
Known as the bnp, base b , to power of p equals number n .

Note If asked to solve an equation with logs we must have the same base

$$\log a + \log b = \log ab$$

$$\log a - \log b = \log \frac{a}{b}$$

$$\log a^n = n \log a$$

$$\log_b a = \frac{\log_x a}{\log_x b}$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

Note We need to know rules backwards.

$$\log ab = \log a + \log b$$

$$\log \frac{a}{b} = \log a - \log b$$

$$n \log a = \log a^n$$

Watch out for simple mistakes that students who don't really know the rules will do

$$(\log a)^n \neq n \log a$$

$$\frac{\log a}{\log b} \neq \log(a - b)$$

Type 1 Logs equal a constant

Step 1 logs to one side numbers to the other.

Step 2 Use rules of logs to have one log = one number.

Step 3 Use definition of logs to get rid of logs and solve.

Note Must check answer because not allowed log of a negative.

Type 2 Logs on both sides.

Step 1 Use rules of logs to have one log = one log.

Step 2 Drop the logs on both sides and solve.

Type 3 Change of base

Note Must change the base and a good rule of thumb that we will use in other parts of the course is **break the bigger one down.**

Type 4 x in power then log both sides

When there is a variable in the power and we cannot use indices then we bring logs in on both sides.