Algebra Proofs

This is the first class on proofs.

These have being worth as little as 10 marks and at most 150 marks. The average mark for proofs is 70 marks per leaving cert.

You need to be able to do each of the following proofs. You must do them as we do the revision. They are really only questions with no figures.

With each proof you must

(i) know how to start them off,
(ii) know the hard bit in the middle,
(iii) know the end bit.

Proof 1 If one root is three times the other in the equation $ax^2 + bx + c = 0$ show that $3b^2 = 16ac$

One root is α the other 3α

Sum of roots $\alpha + 3\alpha = -\frac{b}{a}$

$$4\alpha = -\frac{b}{a} \quad \text{--- (i)}$$

Product of the root $\alpha(3\alpha) = \frac{c}{a}$

$$3\alpha^2 = \frac{c}{a} \quad \text{--- (ii)}$$

Solve the simultaneous equations using substitution

From (i) $\alpha = -\frac{b}{4a}$ sub into (ii)

$$3\left(-\frac{b}{4a}\right)^2 = \frac{c}{a}$$

$$\frac{3b^2}{16a^2} = \frac{c}{a} \qquad \text{cross multiply}$$

$$3ab^2 = 16a^2c \qquad \text{divide across by } a$$

$$3b^2 = 16ac$$

Proof 2 If one root is n times the other in the equation $ax^2 + bx + c = 0$ show that $n^2b = (n+1)^2 ac$

One root is α the other $n\alpha$

$$\alpha + n\alpha = -\frac{b}{a}$$

$$(n+1)\alpha = -\frac{b}{a} - - - (i)$$

$$\alpha(n\alpha) = \frac{c}{a}$$

$$n\alpha^2 = \frac{c}{a} - - - (ii)$$

From (i) $\alpha = -\dfrac{b}{a(n+1)}$

$$\alpha^2 = \frac{b^2}{a^2(n+1)^2} \text{ sub into } \textit{(ii)}$$

$$n\frac{b^2}{a^2(n+1)^2} = \frac{c}{a}$$

$$nab^2 = (n+1)^2 a^2 c$$

$$nb^2 = (n+1)^2 ac$$

Proof 3 If k is a real number such that $f(k) = 0$, prove that $x - k$ is a factor of $f(x) = ax^3 + bx^2 + cx + d$ where $a, b, c, d \in R$. (This is the factor Theorem)

We cannot prove $x - k$ is a factor of $f(x)$ so we first prove $x - k$ is a factor of $f(x) - f(k)$

$$f(x) = ax^3 + bx^2 + cx + d$$
$$f(k) = ak^3 + bk^2 + ck + d$$

Prove that $x - k$ is a factor of $f(x) - f(k)$

$$f(x) - f(k) = ax^3 + bx^2 + cx + d - (ak^3 + bk^2 + ck + d)$$

$$= ax^3 + bx^2 + cx + d - ak^3 - bk^2 - ck - d$$

$$= ax^3 - ak^3 + bx^2 - bk^2 + cx - ck$$

$$= a(x^3 - k^3) + b(x^2 - k^2) + c(x - k)$$

$$= a(x - k)(x^2 + xk + k^2) + b(x - k)(x + k) + c(x - k)$$

$$= (x - k)(ax^2 + akx + ak^2 + bx + bk + c)$$

$$= (x - k)(ax^2 + (ak + b)x + ak^2 + bk + c)$$

Since a, b, c, d and $k \in R$ then $x - k$ is a factor of $f(x) - f(k)$

But $f(k) = 0$ (given at the start) so $x - k$ is a factor of $f(x)$.