Algebra Need To Know 1

Algebra 1

Notation

$3x^2 + 7x + 9$ is called an expression or
a polynomial or
a function.

In the $3x^2$ the 3 is called the <u>coefficient</u> - these are the numbers in front of the letters.

The x is called the <u>variable</u> - these are the letters that stand for different numbers.

The 2 is called the power or <u>indice</u>.

In $3x^2 + 7x + 9$ the 9 is called the <u>constant</u> - these are just numbers with no letters attached

$3x^2 + 7x + 9$ has 3 parts known as 3 <u>terms</u>.

In the number system we must know

N = Natural numbers = whole positive numbers.
Z = Integers = whole positive and negative numbers.
Q = Rational numbers = numbers which can be written as fractions.
R = Real numbers = anything.

Special Polynomials

The next two expressions appear so often that it is vital that you can spot them from any direction.

These are known as perfect squares.

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Square the first, square the second, first by second doubled

Factorisation

Type 1 Factors by Grouping - 2 or 4 terms – take out what's common.

Type 2 Quadratic Factors

Type 3 Difference of two squares

Type 4 Sum and difference of two cubes

Type 5 Combinations

Note x^4 can be written as $(x^2)^2$

Quadratic Equations

These are equations of the form $ax^2 + bx + c = 0$

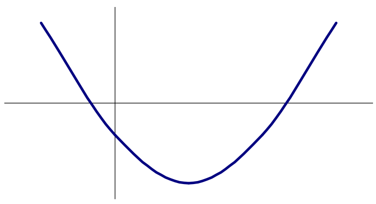
They can be solved in either of two ways

(i) Use the double brackets or guide number and let the factors $= 0$

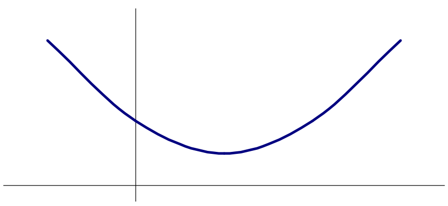
(ii) Use the equation $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac$ is called the discriminate because

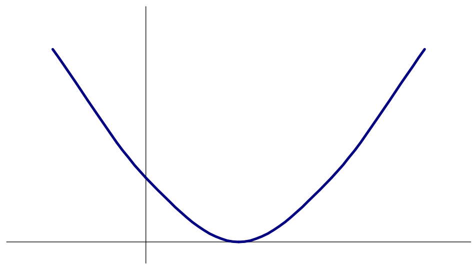
(a) if $b^2 - 4ac > 0$ then there are 2 real roots



(b) if $b^2 - 4ac < 0$ the roots are imaginary (complex roots, unreal roots)



(c) if $b^2 - 4ac = 0$ then there are two equal roots - perfect square



(d) if $b^2 - 4ac \geq 0$ then real roots

Solve equations with substitution.

Algebra 2

Fractions

To Add and Subtract Fractions

Find the common denominator

Equality of Fractions

Find the common denominator

Note Must check both answers to make sure that the bottom-line $\neq 0$

Complex Fractions

When dealing with complex fractions get a common denominator for the top expression and then for the bottom expression so that we end up with just one fraction over one fraction. Then turn the second fraction up - side - down and multiply.

Simultaneous Equations

Type 1 The intersection of a pair of lines.

Type 2 The intersection of a line and curve

Type 3 Where a substitution is needed.

Type 4 A 3 by 3 system of Simultaneous Equations.

Type 5 Equations that become simultaneous

Let coefficients of x on both side be equal and let constants on both sides be equal

Algebra 3

Roots of a Quadratic Equations

Note When we deal with the following the coefficient of x^2 must be 1, so if there is number in front of the x^2, we will divide the whole line by this number.

There follows three ways of doing the same thing

(i) A quadratic equation can be formed using

$$x^2 - (\text{sum of roots})x + \text{product of the roots} = 0$$

(ii) Sum of roots are equal to minus the coefficient of x

Product of roots is equal to the constant.

(iii) Given α and β are the roots of $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

Note We started with $ax^2 + bx + c = 0$ but since the coefficient of x^2 must be 1 we must divide across by a we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Type 1 To form a quadratic given the roots

Type 2 Evaluation questions

Here is a list of common questions that come up when we use α and β

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	Learn.
$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ or $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$	Learn.
$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$	Common denominator.
$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$	Common denominator.
$(\alpha + 2)(\beta + 2) = \alpha\beta + 2(\alpha + \beta) + 4$	Multiply out.
$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$	Factorise.
$\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$	Factorise.

$$\begin{aligned}(\alpha - \beta)^2 &= \alpha^2 - 2\alpha\beta + \beta^2 \\ &= \alpha^2 + \beta^2 - 2\alpha\beta \\ &= (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta \\ &= (\alpha + \beta)^2 - 4\alpha\beta\end{aligned}$$

Learn.

Type 3 Questions where the roots are related

Type 4 To form a quadratic which roots are related to a given quadratic

Algebra 4

Absolute value

$|x|$ means x is always positive. $|x| = \pm x$

Method 1 Split the question up using + and then -

Method 2 Square both sides

Inequalities

We deal with inequalities the same way as equalities.
When we multiply or divide by a minus the inequality sign must change sign.

Note Since the question says $x \in N$ we are only allowed whole positive results.

Double inequalities

When there are two inequality signs in the question split the question into two

Quadratic Inequalities

To solve $ax^2 + bx + c < 0$ let $ax^2 + bx + c = 0$ first.

There is a quick way to finish the question after we have solved the quadratic.

When the coefficient of x^2 is positive put the smaller number first, the bigger number second, the x in the middle and use the last inequality sign.

Note In order to get full marks must draw out a rough graph.

Inequalities and fractions

When the denominator includes a variable the method is always the same. Multiply both sides by the denominator squared.

Note The answer has $x > -1$ and not $x \geq -1$ since we are told in the question $x \neq -1$

Inequalities and absolute values

When we have an inequality and modulus the method is to square both sides

Inequalities proofs

Type 1 Questions, which will use perfect squares backwards

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Note To finish the question off must state that any number squared is positive.

Note Did not have to finish the question since done already in the first part.

To complete the square do the following:

Step 1 Insure the coefficient of x^2 is 1.
Step 2 Take half the coefficient of x and square.
Step 3 Write as a perfect square.

Type 2 Inequalities where we need to use the given information.