Circle 5

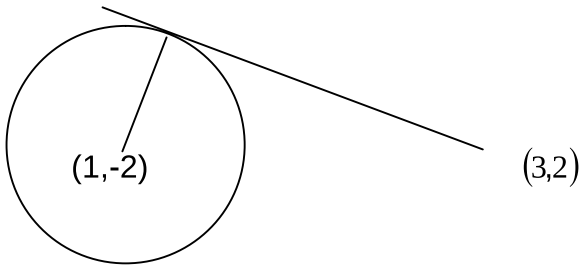
In this class we will deal with tangents from a point outside a circle, common chords and tangents, touching circles.

Tangents from a point outside a circle

Example 1 Find the equation of the tangents from the point $(3,2)$ to the circle

$$x^2 + y^2 - 2x + 4y - 11 = 0$$

From the circle we get centre of $(1,-2)$ and radius 4



Let the line be $y - y_1 = m(x - x_1)$

Sub in the point $(3,2)$

$$y - 2 = m(x - 3)$$

$$y - 2 = mx - 3m$$

$$mx - y - 3m + 2 = 0$$

Perpendicular distance from centre $(1,-2)$ to the line $mx - y - 3m + 2 = 0$ is equal to the radius 4.

$a = m,\ b = -1,\ \ c = 2 - 3m, x_1 = 1, y_1 = -2$

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$\left|\frac{m+2+2-3m}{\sqrt{m^2+1}}\right| = 4$$

$$|4-2m| = 4\sqrt{m^2+1}$$

$$|2-m| = 2\sqrt{m^2+1}$$

$$m^2-4m+4 = 4m^2+4$$

$$3m^2-4m = 0$$

$$m = 0 \quad \text{or} \quad m = \frac{4}{3}$$

Using both slope and the point $(3,2)$ in the equation of the line $y - y_1 = m(x - x_1)$

Tangents are $y = 2$ and $4x - 3y = 6$

Common Chords and Tangents

If $S = x^2 + y^2 + 2gx + 2fy + c = 0$ and $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ represent two circles then the equation

$S - S_1 = 0$ represent a straight line which is the

(a) common chord if $S = 0$ and $S_1 = 0$ have two points of intersection.

(b) common tangent at the point of intersection if $S = 0$ and $S_1 = 0$ have only one point of intersection i.e. if they are touching.

Example 2 Find the equation of the common chord of the circles $x^2+y^2+6x-6y-8=0$ and $x^2+y^2-4x+4y-8=0$ and hence find the points of intersection.

Equation of chord is given by

$$x^2+y^2+6x-6y-8-\left(x^2+y^2-4x+4y-8\right)=0$$

$$x^2+y^2+6x-6y-8-x^2-y^2+4x-4y+8=0$$

$$10x-10y=0$$

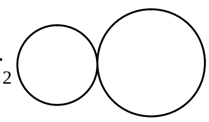
$$x-y=0$$

To find the points of intersection use simultaneous equations between the circle $x^2+y^2+6x-6y-8=0$ and the line $x-y=0$ to find the points $(2,2)$ and $(-2,-2)$

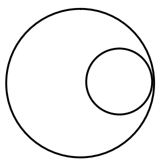
Touching circles

Let two circles S_1 and S_2 have radii of r_1 and r_2 respectively and let d be the distance between their centres then

The circles can touch externally if $d=r_1+r_2$



The circles can touch internally if $d=r_1-r_2$



Example 3 Prove that the circles $x^2 + y^2 - 2x + 4y - 8 = 0$ and $x^2 + y^2 + 6x - 8y + 12 = 0$ touch each other and find the point of contact.

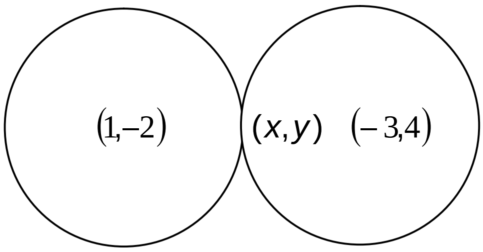
$x^2 + y^2 - 2x + 4y - 8 = 0$
Centre $(1,-2)$ radius $= \sqrt{13}$

$x^2 + y^2 + 6x - 8y + 12 = 0$
Centre $(-3,4)$ radius $= \sqrt{13}$

Distance from $(1,-2)$ to $(-3,4)$ $= \sqrt{52} = 2\sqrt{13}$

$2\sqrt{13} = \sqrt{13} + \sqrt{13}$ so circle touch externally.

Draw a diagram to see what is happening



Required point is the midpoint of $(1,-2)$ and $(-3,4)$
Answer is $(-1,1)$