

Complex Numbers

Class 1

Example 1 Solve the equation $z^2 - 4z + 5 = 0$.

Example 2 Given that $i^2 = -1$, find the value of

(a) i^9

(b) i^{10}

Example 3 If $z_1 = 2 + 4i$ and $z_2 = 3 - 5i$ find the values of

(i) $z_1 + z_2$

(ii) $2z_1 - 3z_2$

Example 4 If $z = 3 - i$ and $w = 5 + 2i$ simplify each of the following

(i) iz

(ii) zw

Example 5 If $z = 3 + 4i$ find the value of $z\bar{z}$.

$$z = 3 + 4i \text{ then } \bar{z} = 3 - 4i$$

Example 6 Write $\frac{z_1}{z_2}$ in the form $a + bi$ where $z_1 = 4 + 3i$ and $z_2 = 2 - 5i$

Example 7 If $p(2 + 3i) + q(-3 + i) = -4 + 5i$ find the value of p and the value of q where $p \in R$ and $q \in R$

Example 8 Let $z = 1 + i$ and let \bar{z} be the conjugate of z .

Express $\frac{z}{\bar{z}}$ in the form $x + yi$, $x, y \in R$.

Example 9 Plot on the argand diagram z_1 , \bar{z}_2 and $z_1 - \bar{z}_1$. Find the image of z_1 under central symmetry in the origin where $z_1 = 4 + 3i$ and $z_2 = 2 + 5i$

Class 2

Example 1 Find $|z_1|$ and $|z_2|$ hence show that $|z_1 z_2| = |z_1| |z_2|$ where $z_1 = 3 - 5i$ and $z_2 = 1 - 2i$

Example 2 If $z = 4 + ki$ and $|z| = 5$ find two values of k where $k \in R$.

Example 3 If $z = 2 + i$ and $w = 3 - 6i$ find the value of k where $k \in R$ if $|w| = k|z|$

Example 4 Prove that $|z| = |\bar{z}|$

Example 5 Find $\sqrt{-3 + 4i}$

Example 6 Given that $\sqrt{-3 + 4i} = \pm(1 + 2i)$ solve $z^2 - (4 - 2i)z + 6 - 8i = 0$

Example 7 Form a quadratic with roots $5 \pm 3i$

Example 8 Show that $3 + 5i$ is a root of $z^2 - 6z + 34 = 0$ and state the other root.

Example 9 If $1 + 3i$ is a root of $x^2 + hx + ki = 0$ find the value of h and the value of k .

Example 10 If $2 + 3i$ is a root of $x^2 - (5 - i)x + p = 0$ find the other root and find the value of p

Class 3

Example 1 Given that $P(z) = z^3 - kz^2 + 22z - 20 = 0$, $k \in \mathbb{R}$

$3 + i$ is one root of $P(z) = 0$. Find the value of k and find the other two roots.

Example 2 Represent $1 + \sqrt{3}i$ in polar form.

Example 3 Given $z_1 = 4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$ and $z_2 = 10\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

Class 4

Example 1 If $z = 2 - 2i$ express z^4 in the form $a + bi$

Example 2 Prove $\sin 4\theta = 4 \sin \theta (2 \cos^3 \theta - \cos \theta)$

Example 3 If $z = (-2 - 2\sqrt{3}i)^{\frac{3}{2}}$ find two values of z in the form $a + bi$

Example 4 If $z^3 = 1$ use De Moivre's Theorem to find the cube roots.

Example 5 Given that 1 , ω and ω^2 are the cube roots of unity

- (i) Prove that $1 + \omega + \omega^2 = 0$
- (ii) Evaluate $(1 - \omega - \omega^2)^3$

Example 6 Prove $\cos^3 \theta = \frac{1}{4}(\cos 3\theta + 3 \cos \theta)$

Matrices

Example 1 Find $A + B$ if $A = \begin{pmatrix} 5 & 1 \\ 9 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 8 & -3 \end{pmatrix}$

Example 2 Evaluate $3 \begin{pmatrix} 2 & -3 \\ 4 & 9 \end{pmatrix}$

Example 3 Evaluate each of the following

Example 4 If $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ evaluate $A^2 - 3A + 4I$.

Example 5 Find the inverse of $A = \begin{pmatrix} 3 & -2 \\ 4 & 5 \end{pmatrix}$

Example 6 If $\begin{pmatrix} 4 & -6 \\ 2 & k \end{pmatrix}$ is singular find the value of k .

Example 7 Solve for M the equation $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} M = \begin{pmatrix} 5 & -2 \\ 0 & 4 \end{pmatrix}$

Example 8 Solve $2x - y = 5$ and $3x + y = 5$ using matrices.

Example 9 Prove $(P^{-1}MP)^3 = P^{-1}M^3P$

Example 10 If $N = \begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix}$ and $P = \begin{pmatrix} 3 & -1 \\ 1 & 3 \end{pmatrix}$ find $P^{-1}NP$ and hence find N^{20} .

Proofs Class

Proof 1 If $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ to find an expression for

(i) $z_1 z_2$

(ii) $\frac{z_1}{z_2}$

(iii) $\frac{1}{z_1}$

Proof 2 Proof of De Moivre's Theorem.

If $z = r(\cos \vartheta + i \sin \vartheta)$ prove $z^n = r^n(\cos n\vartheta + i \sin n\vartheta)$