Functions 2

This is where we need to find one or two unknowns.

Functions with One Unknown

Example 1 Let f be the function $f : x \to 3x - 2$ find k if $f(k) = 19$ where $k \in R$

The method does not change.

$$f(x) = 3x - 2$$

$$f(k) = 3k - 2 = 19$$

$$3k = 21$$

$$k = 7$$

Example 2 If $g(x) = 2x - 5$ find k if $g(k+1) = 19$, where $k \in R$.

$$g(x) = 2x - 5$$

$$g(k+1) = 2(k+1) - 5 = 19$$

$$2k + 2 - 5 = 19$$

$$2k - 3 = 19$$

$$2k = 22$$

$$k = 11$$

Example 3 If $f(x) = 4x + b$ find the value of b given that $f(2) = 11$, where $b \in R$

$$f(x) = 4x + b$$

$$f(2) = 4(2) + b = 11$$

$$8 + b = 11$$

$$b = 3$$

Example 4 Given that $g(x) = ax + 5$ find the value of a given that $g(2) = -9$, where $a \in R$

$$g(x) = ax + 5$$

$$g(2) = 2a + 5 = -9$$

$$2a = -14$$

$$a = -7$$

Example 5 If $f(x) = 2x - 9$ find the value of k if $f(0) = k[f(3)]$, where $k \in R$.

Need to find $f(0)$ and $f(3)$ and then form an equation to find k.

$$f(x) = 2x - 9$$

$$f(0) = 2(0) - 9$$

$$= 0 - 9 = -9$$

$$f(x) = 2x - 9$$

$$f(3) = 2(3) - 9$$

$$= 6 - 9 = -3$$

$$f(0) = k[f(3)]$$

$-9 = -3k$ substitute $f(0) = -9$ and $f(3) = -3$

$9 = 3k$ change sign of both sides

$$3k = 9$$

$$k = 3$$

Two Unknowns

When there are two unknowns the solution will nearly always involve solving simultaneous equations from algebra.

Example 6 If $f(x) = ax + b$ find the value of a and b given that $f(2) = 7$ and $f(3) = 13$

$$f(x) = ax + b$$

$$f(2) = 2a + b = 7$$

$$f(3) = 3a + b = 13$$

Use simultaneous equations to find $a = 6$ and $b = -5$.

Example 7 $f(x) = ax^2 + bx - 8$, where a and b are real numbers.

If $f(1) = -9$ and $f(-1) = 3$, find the value of a and the value of b.

$$f(x) = ax^2 + bx - 8$$

$$f(1) = a(1)^2 + b(1) - 8 = -9$$

$$a + b - 8 = -9$$

$$a + b = -1$$

$$f(x) = ax^2 + bx - 8$$

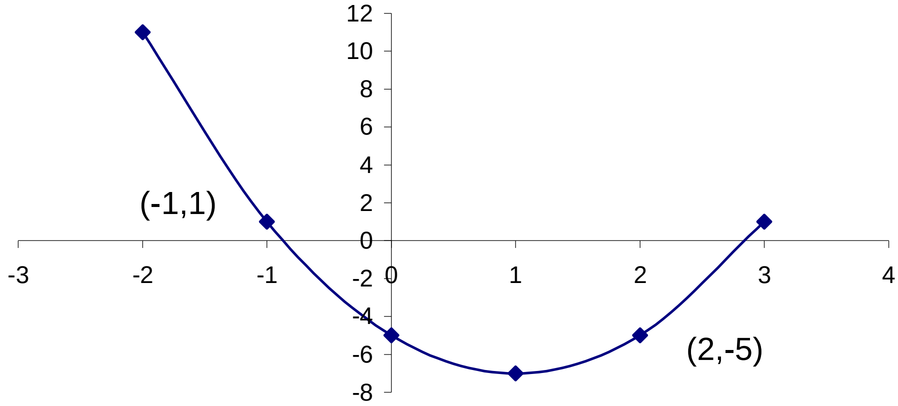
$$f(-1) = a(-1)^2 + b(-1) - 8 = 3$$

$$a(1) - b - 8 = 3$$

$$a - b = 11$$

Use simultaneous equation to find $a = 5$ and $b = -6$

Example 8 If $g(x) = 2x^2 + ax + b$ find the value of a and the value of b.



From the diagram we know two points are (-1,1) and (2,-5). What use can we make from this?

(-1,1) means when $x = -1$ then $y = g(x) = 1$

$$y = 2x^2 + ax + b$$

$$1 = 2(-1)^2 + a(-1) + b$$

$$1 = 2 - a + b$$

$$a - b = 1$$

(2,-5) means when $x = 2$ then $y = g(x) = -5$

$$y = 2x^2 + ax + b$$

$$-5 = 2(2)^2 + a(2) + b$$

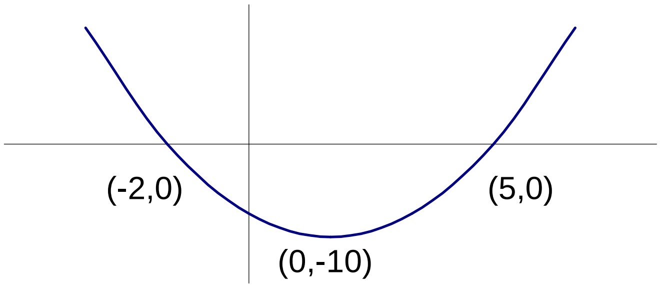
$$-5 = 8 + 2a + b$$

$$-2a - b = 13$$

$$2a + b = -13$$

Use simultaneous equations to find answers of $a = -4$ and $b = -5$.

Example 9 If $f(x) = ax^2 + bx + c$ find the values of a, b and c.



From the diagram we know three points are (-2,0) and (5,0) and (0,-10)

$$f(x) = ax^2 + bx + c$$

$$y = ax^2 + bx + c$$

(0,-10) is the best point to start with. It means when $x = 0$ then $y = -10$

$$y = ax^2 + bx + c$$
$$-10 = a(0)^2 + b(0) + c$$
$$c = -10$$

(-2,0) means when $x = -2$ then $y = 0$

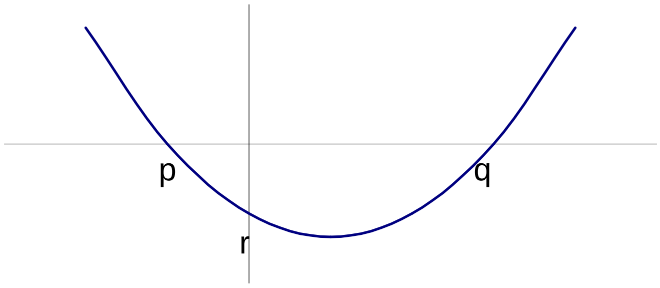
$$y = ax^2 + bx + c$$
$$0 = a(-2)^2 + b(-2) + c \quad \text{but } c = -10$$
$$0 = 4a - 2b - 10$$
$$-4a + 2b = -10$$
$$2a - b = 5$$

(5,0) means when $x = 5$ then $y = 0$

$$y = ax^2 + bx + c$$
$$0 = a(5)^2 + b(5) + c \quad \text{but } c = -10$$
$$0 = 25a + 5b - 10$$
$$-25a - 5b = -10$$
$$5a + b = 2$$

Use simultaneous equations to find answers of $a = 1$ and $b = -3$.

Example 10 The curve $f(x) = x^2 - 2x - 3$ is as shown find the points p, q and r.



The points p and q are where the graph cuts the x - axis.

$$f(x) = x^2 - 2x - 3$$
$$y = x^2 - 2x - 3$$

x - axis then $y = 0$

$$x^2 - 2x - 3 = 0$$
$$x^2 - 3x + x - 3 = 0$$
$$x(x-3) + 1(x-3) = 0$$
$$(x+1)(x-3) = 0$$

$$x + 1 = 0 \qquad\qquad x - 3 = 0$$
$$x = -1 \qquad\qquad x = 3$$

Point p is (-1,0) and q is (3,0)

The point r is where the curve cuts the y axis so $x = 0$.

$$y = x^2 - 2x - 3$$
$$y = 0^2 - 2(0) - 3$$
$$y = -3$$

Point r is (0,-3)