

Graphs 2

This class will go through graphs of quadratics and straight lines and practical problems.

This continues from where we left the last class.

Graph of quadratics and Straight lines.

Example 1 Using the same axes and the same scales, draw a graph of

$$f(x) = x^2 + 3x - 2$$

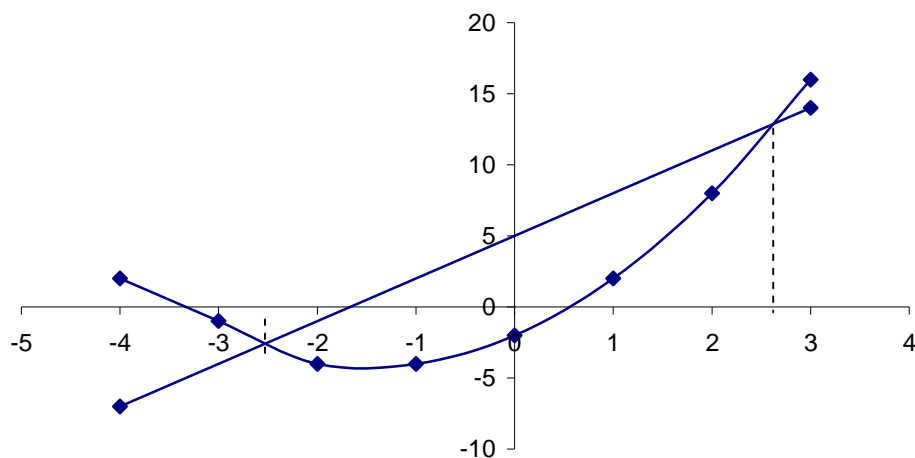
$$g(x) = 3x + 5 \text{ in the domain } -4 \leq x \leq 3, x \in R.$$

x	-4	-3	-2	-1	0	1	2	3
x^2	16	9	4	1	0	1	4	9
$3x$	-12	-9	-6	-3	0	3	6	9
-2	-2	-2	-2	-2	-2	-2	-2	-2
$f(x) = y$	2	-2	-4	-4	-2	2	8	16

The second graph is a line, since there is no x^2 so we only need two points.

We make out a table but we just use the first and the last values of x .

x	-4	3
$3x$	-12	9
5	5	5
$f(x) = y$	-7	14



Type 9 To find values of x for which $f(x) = g(x)$.

What they really want is the values of x for which the curve equals the line. We need the points of intersection of the curve and line.

Come straight up and down from points where the graphs intersect.

Answers $x = -2.5$ and $x = 2.6$.

Type 10 To find values of x for which $f(x) < g(x)$.

This time we want the values of x for which the curve is below the line.

Answers From -2.5 to 2.6 . Which could be written $x > -2.5$ and $x < 2.6$ or written as $-2.5 < x < 2.6$

Practical Problems

Example 2 A rectangle has one-side x meters long. The rectangle has a total perimeter of 10 meters. Show that the area of the rectangle is $x(5 - x)$.

Draw the graph of the function

$$f(x) = 5x - x^2 \text{ in the domain } 0 \leq x \leq 5, x \in R.$$

Use your graph to find

- (i) the width of the rectangle when the area is $4.7m^2$
- (ii) the maximum area of the rectangle
- (iii) the dimensions of the maximum area
- (iv) the area when the width is $4.5m$.

To answer the first part draw a diagram of a rectangle, where one side is x



The total perimeter is 10. The 4 sides add up to 10 so that two sides add to 5.

We know one side is x so the other side must be $5 - x$.

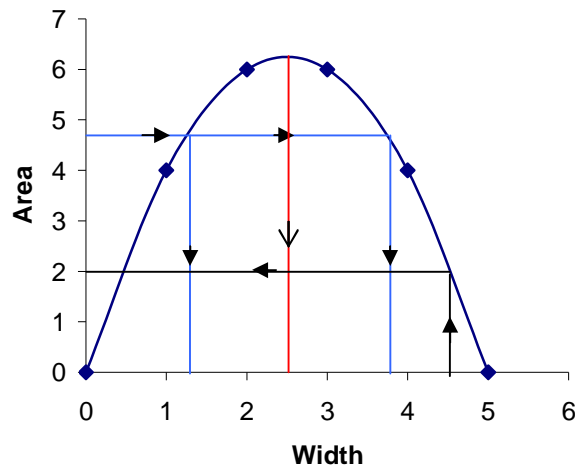
Area of a rectangle = length \times breadth

$$= x(5 - x)$$

Draw the graph of the function $f(x) = 5x - x^2$ in the domain $0 \leq x \leq 5$.

x	0	1	2	3	4	5
$5x$	0	5	10	15	20	25
$-x^2$	0	-1	-4	-9	-16	-25
$f(x) = y$	0	4	6	6	4	0

When we have the diagram drawn the main thing for you to see is that the graph has the same equation as the area of the rectangle. We can replace $f(x)$ for area. Area is put on the y axis and width on the x axis.



With any question on the graph we must decide which axes to start on. We must decide have they given us the area (so that we will start on the vertical axis, go across to hit the graph and the down), or width (so that we start on the horizontal axis, go up to hit the graph and then across).

Note Up to now each question has being answered on different diagrams but in the exam you draw just one diagram on which you show all your answers as with this question. A big clear neat diagram is therefore vital.

- (i) The width of the rectangle when the area is 4.7. Start at 4.7 on the vertical axis across and the down to find the two widths.

Answers of 1.2 and 3.8

(ii) The maximum area of the rectangle.

This is the highest point reached by the graph. Answer 6.25

(iii) The dimensions of the maximum area.

Come straight down from the maximum point.

Answer Width $x = 2.5$ and length $5 - x = 2.5$

(iv) The area when the width is 4.5.

This time we know the width is 4.5 so start at 4.5 on the x axis go up and then across to find the area.

Answer Area = 2.1

Example 3 The height of a missile above the ground x seconds after being fired is given by $f(x)$, where

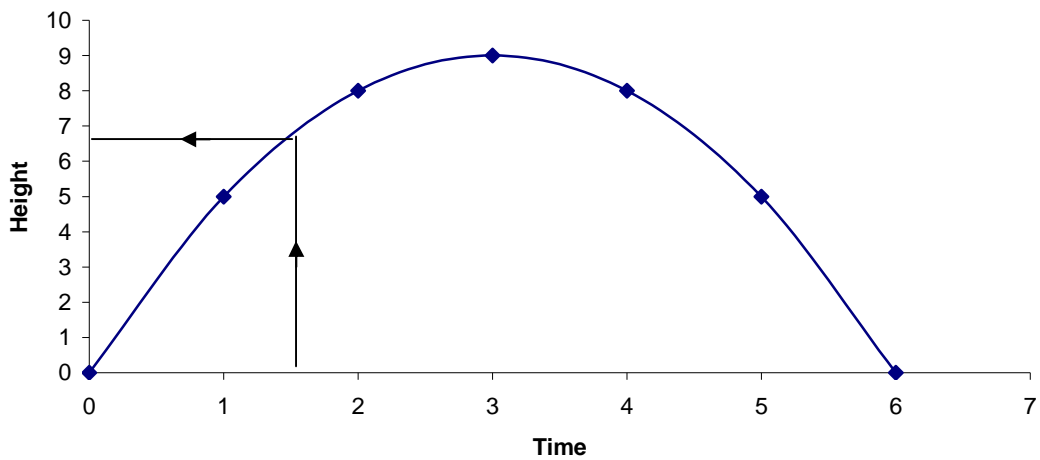
$$f(x) = 6x - x^2 \text{ in the domain } 0 \leq x \leq 6, x \in R.$$

Draw a graph of $f(x)$ and use it to find

- (i) the height of the missile after 1.5 seconds
- (ii) the maximum height reached by the missile
- (iii) the time taken to reach the maximum height
- (iv) the time the missile is over the ground.

x	0	1	2	3	4	5	6
$6x$	0	6	12	18	24	30	36
$-x^2$	0	-1	-4	-9	-16	-25	-36
$f(x) = y$	0	5	8	9	8	5	0

This time we replace $f(x)$ for height. Height is put on the y axis and time on the x axis.



(i) The height of the missile after 1.5 seconds.

Start on x axis at 1.5 go up and then across to find the height of 6.8.

(ii) The maximum height reached by the missile.

This is the highest point on the curve. Height = 9.

(iii) The time taken to reach the maximum height.

Come straight down from the highest point. Answer of 3 seconds.

(iv) The time the missile is over the ground.

This is the period the curve is above the x axis. The answer is 6 seconds.