The Line 1

This class contains the junior cert material you are supposed to know.

Junior Cert Revised

A point is named using lower case letters and a line is named using capital letters.

With each one of the formulae we must follow the steps

Step 1 Write down the points from the question.
Step 2 Label the points (x_1, y_1) and (x_2, y_2).
Step 3 Write down the formula.
Step 4 Put the figures into the formula and work it out.

Distance

Distance between two points (x_1, y_1) and (x_2, y_2) is given by

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 1 a(3,-1) and b(-5,7) are two points find $|ab|$.

Note $|ab|$ means the distance from a to b.

$$\begin{array}{cc} (3,-1) & (-5,7) \\ (x_1, y_1) & (x_2, y_2) \end{array}$$

$$\begin{aligned} &\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5-3)^2 + (7-(-1))^2} \\ &= \sqrt{(-8)^2 + (8)^2} \\ &= \sqrt{64+64} \\ &= \sqrt{128} \\ &= \sqrt{64}\sqrt{2} \\ &= 8\sqrt{2} \end{aligned}$$

Example 2 Two point are $a(5,2)$ and $b(8,k)$ such that $|ab| = 5$. Find the values of $k \in R$.

$$
\begin{array}{cc}
(5,2) & (8,k) \\
(x_1,y_1) & (x_2,y_2)
\end{array}
$$

$$
\begin{aligned}
&\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&\sqrt{(8-5)^2 + (k-2)^2} = 5 \\
&\sqrt{3^2 + (k-2)^2} = 5 \\
&\sqrt{9 + k^2 - 4k + 4} = 5 \\
&\sqrt{k^2 - 4k + 13} = 5 \\
&k^2 - 4k + 13 = 25 \\
&k^2 - 4k - 12 = 0 \\
&(k-6)(k+2) = 0
\end{aligned}
$$

$$k = 6 \text{ or } k = -2$$

Here is another nice way to finish off the same question.

$$
\begin{aligned}
\sqrt{3^2 + (k-2)^2} &= 5 \\
9 + (k-2)^2 &= 25 \\
(k-2)^2 &= 16 \\
k - 2 &= \pm 4
\end{aligned}
$$

$$
\begin{array}{ll}
k - 2 = 4 & k - 2 = -4 \\
k = 6 & k = -2
\end{array}
$$

Midpoint

The midpoint between two given points.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example 3 Find the midpoint of (-3,4) and (9,-8)

Answer is $(3,-2)$

Example 4 If $a(3,7)$ is the midpoint of $[pq]$ where $p(6,5)$ find q.

If we read the question carefully we see that $a(3,7)$ is in the centre of $p(6,5)$ and q so that if we look solely at the x coordinates and then at the y coordinate.

x coordinates	6	down 3 to	3	down 3 to	0
y coordinates	5	up 2 to	7	up 2 to	9

Answer therefore is (0,9)

Area of a triangle

The area of a triangle where one of the vertices must be (0,0) is

$$\text{Area} = \frac{1}{2}\left|x_1 y_2 - x_2 y_1\right|$$

Note The two lines either side $|\;|$ stand for absolute value.

Note Vertex is the corner point. A triangle has 3 vertices.

Example 5 Find the area of (-2,4), (1,-3),and (6,-2)

We must move one of the points onto (0,0) by using a translation. This means we look at the x and then the y coordinates and see by how much they have changed. We make this into a rule, which we then apply to the other points.

The x values go from –2 to 0 so they have gone up 2.

The y values have gone from 4 to 0 so the have gone down by 4.

(-2,4) ⟶ (0,0) x up by 2, y down by 4
(1,-3) ⟶ (3,-7) x up by 2, y down by 4
(6,-2) ⟶ (8,-6) x up by 2, y down by 4

(3,-7) and (8,-6)
(x_1, y_1) (x_2, y_2)

$$\text{Area} = \frac{1}{2}|x_1y_2 - x_2y_1|$$
$$= \frac{1}{2}|3(-6) - 8(-7)|$$
$$= \frac{1}{2}|-18 + 56|$$
$$= \frac{1}{2}|38| = 19 \text{ sq units}$$

Example 6 $a(-1,3)$, $b(1,-1)$ and $c(-3,k)$ are the vertices of a triangle abc. If the area of the triangle is 10 find two possible values of $k \in R$.

(-1,3)	⟶	(0,0)	x up by 1, y down by 3
(1,-1)	⟶	(2,-4)	x up by 1, y down by 3
$(-3,k)$	⟶	$(-2,k-3)$	x up by 1, y down by 3

(2,-4) and $(-2,k-3)$

(x_1, y_1) (x_2, y_2)

$$\text{Area} = \frac{1}{2}|x_1y_2 - x_2y_1|$$
$$\frac{1}{2}|2(k-3)-(-2)(-4)| = 10$$
$$\frac{1}{2}|2k - 6 - 8| = 10$$
$$\frac{1}{2}|2k - 14| = 10$$
$$|k - 7| = 10$$
$$k - 7 = \pm 10$$

the question splits into 2 parts one with the + and other with the -

$k - 7 = 10$ $\qquad$ $k - 7 = -10$

$k = 17$ $\qquad$ $k = -3$

Slope between two points

Slope of a line between two given points is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 7 Find slope between $a(2,-3)$ and $b(3,-1)$

$(x_1, y_1) \quad (x_2, y_2)$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1-(-3)}{3-2} \\ &= \frac{-1+3}{1} = 2 \end{aligned}$$

Note Always leave slope as a whole number or a fraction

Slope of a given line

The slope of a given line $ax + by + c = 0$ is given by

$$m = -a/b.$$

Note To find the slope of a line it comes down to

$$\text{minus} \frac{\text{number infront of } x}{\text{number infront of the } y}$$

Note There are other ways of finding the slope of a curve,

$m =$ tan of angle with positive sense of x axis.

$m = \frac{dy}{dx} =$ slope of tangent at the point (x, y)

Parallel and Perpendicular Lines

If two lines are parallel $M \| L$ then the slopes are equal then $m_1 = m_2$

If two lines are perpendicular $L \perp M$ then $m_1 m_2 = -1$

Note If we are given a slope and asked to find a parallel slope then answer is the exact same as first slope.

Note If we are given a slope and asked to find a perpendicular slope then answer is got by inverting and changing sign of the original slope.

Example 8 If the line L has the following slope find the slope of the lines parallel and perpendicular to L.

If L has slope $m = \frac{2}{3}$ then parallel slope is $m = \frac{2}{3}$ but perpendicular slope is $m = -\frac{3}{2}$

If L has slope $m = 6$ then parallel slope is $m = 6$ but perpendicular slope is $m = -\frac{1}{6}$

Example 9 L is the line $3x - 7y = 13$ and M is the line $7x + 3y = 21$ prove that $L \perp M$.

Slope of L is $m_1 = -\frac{3}{-7} = \frac{3}{7}$

Slope of M is $m_2 = -\frac{7}{3}$

$L \perp M$ since $m_1 m_2 = \left(\frac{3}{7}\right)\left(-\frac{7}{3}\right) = -1$