The Line 4

This class contains perpendicular distance formula and some words of importance.

Perpendicular Distance

This is the distance from a point to a line.

The perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is given by

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Note It is very important that before we use the formula that we have the line written in the correct form i.e. with $ax + by + c = 0$.

Write down clearly the vales of a, b, c, x_1 and y_1.

Type 1 Distance from a point to a line:

Example 1 Find the distance from (2,-3) to $x + 5y - 4 = 0$

$$a = 1,\ b = 5,\ c = -4,\ x_1 = 2,\ y_1 = -3$$

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{1(2) + 5(-3) - 4}{\sqrt{1^2 + 5^2}}\right|$$

$$= \left|\frac{-17}{\sqrt{26}}\right| = \frac{17}{\sqrt{26}}$$

Type 2 Given the distance and one unknown

Example 2 If the distance from (1,2) to $3x+4y+k=0$ is 6 find two possible values of k.

We must use the perpendicular formula even though it is not mentioned in the question as we are given the distance from a point to a line.

$a=1,\ b=4,\ c=k,\ x_1=3,\ y_1=2$ use the formula and you cannot go wrong.

$$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$\left|\frac{1(3)+4(2)+k}{\sqrt{3^2+4^2}}\right|=6$$

$$\left|\frac{11+k}{5}\right|=6$$

$$|11+k|=30$$

$$11+k=\pm 30$$

$k=19$ or $k=-41$

Type 3 Distance between two parallel lines.

Example 3 Find the distance between the lines $3x+4y-1=0$ and $3x+4y-15=0$

Parallel lines. A point on $3x+4y-15=0$ is (5,0)

The perpendicular distance from (5,0) to $3x+4y-1=0$

$a=3,\ b=4,\ c=-1,\ \ x_1=5,\ y_1=0$

$$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{3(5)+4(0)-1}{\sqrt{3^2+4^2}}\right|$$

$$=\left|\frac{14}{\sqrt{25}}\right|=\frac{14}{5}$$

Type 4 Given a point on a line and the perpendicular distance to another point.

Example 4 Find the equation of the two lines through the point (-3,0), which are at a distance of $\sqrt{5}$ from the (4,1).

2 equations needed here

(i) Equation of a line L in the form $y-y_1=m(x-x_1)$

(ii) Perpendicular distance formula

(-3,0) is on L so

$$y-0=m(x+3)$$
$$y=mx+3m$$
$$mx-y+3m=0$$

Now perpendicular distance from (4,1) to L is $\sqrt{5}$.

$a=m,\ b=-1,\ c=3m,\ x_1=4,\ y_1=1$

$$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$\left|\frac{m(4)-1(1)+3m}{\sqrt{m^2+(-1)^2}}\right|=\sqrt{5}$$

$$\left|\frac{7m-1}{\sqrt{m^2+1}}\right|=\sqrt{5}$$

$$|7m-1|=\sqrt{5}\sqrt{m^2+1}$$

$$49m^2-14m+1=5m^2+5$$

$$44m^2-14m-4=0$$

$$22m^2-7m-2=0$$

$$(11m+2)(2m-1)=0$$

$$m=-\frac{2}{11} \text{ or } m=\frac{1}{2}$$

Use the equation of the line $y-y_1=m(x-x_1)$ with the two slopes and the point (-3,0)

Two answers $2x+11y=-6$ and $x-2y=-3$

Example 5 If p is the length of the perpendicular from the origin to the line

$$\frac{x}{a}+\frac{y}{b}=1,$$

prove that

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

We need to find the perpendicular distance form (0,0) to

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{a}+\frac{y}{b}=1$$

$$bx+ay-ab=0$$

$$a=b,\ b=a,\ c=-ab,\ x_1=0,\ y_1=0$$

$$\mathrm{p}=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$p=\left|\frac{b(0)+a(0)-ab}{\sqrt{a^2+b^2}}\right|$$

$$p=\left|\frac{-ab}{\sqrt{a^2+b^2}}\right|$$

$$p^2=\frac{a^2b^2}{a^2+b^2}$$

From the question we are told

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2}$$

$$p^2 = \frac{a^2 b^2}{a^2 + b^2}$$

Some words that may appear in questions

<u>Collinear</u> 3 or more points on the same straight line - show the slope between them is the same.

<u>Centroid</u> point of intersection of *medians* (line from vertices to midpoint of opposite line)-use formula

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

<u>Circumcentre</u> point of intersection of the *mediators* (perpendicular bisector of the sides of a triangle)

<u>Orthocentre</u> point of intersection of the lines from the vertices perpendicular to the opposite sides of a triangle.