Functions and Graphs Need to Know

Functions 1

Function Questions

$f(x) = y$. We can put the couple together and show values together on a graph.

Domain = set of first elements of a couple.

Range = set of second elements of a couple.

Note $f(x)$ is the same as $g(x)$ is the same as $h(x)$.

Note Two things to remember here are

(i) Square first then multiply

(ii) Any number squared is positive.

Functions 2

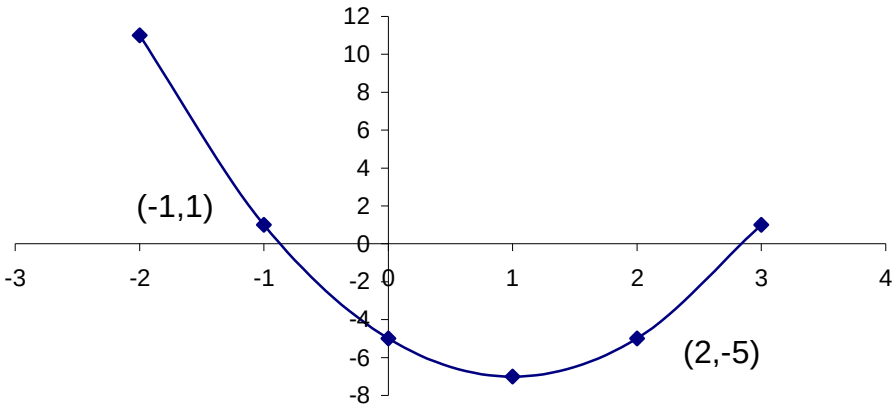
Functions with One Unknown

Do what you are told in the question.

Two Unknowns

When there are two unknowns the solution will nearly always involve solving simultaneous equations from algebra.

If $g(x) = 2x^2 + ax + b$ find the value of a and the value of b.



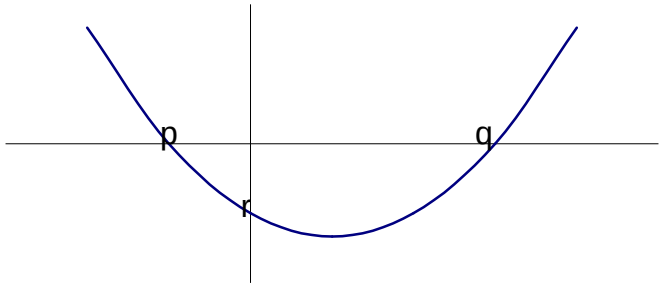
From the diagram we know two points are (-1,1) and (2,-5). What use can we make from this?

(-1,1) means when $x = -1$ then $y = g(x) = 1$

(2,-5) means when $x = 2$ then $y = g(x) = -5$

Use simultaneous equations to find answers of $a = -4$ and $b = -5$.

The curve $f(x) = x^2 - 2x - 3$ is as shown find the points p, q and r.



The points p and q are where the graph cuts the x - axis.

x - axis then $y = 0$

The point r is where the curve cuts the y axis so $x = 0$.

Graphs 1

Note All diagrams are drawn on graph paper using a pencil.

Graph of two lines

To find where a line cuts the x - axis put $y = 0$ into the equation and get a value for x.

To find where a line cuts the y - axis put $x = 0$ into the equation and get a value for y.

Note The points on the lines are joined using a ruler.

Graph of quadratics.

Draw the graph of $f(x) = 2x^2 - 2x - 3$ in the domain $-2 \le x \le 3$, $x \in R$.

'In the domain $-2 \le x \le 3$' means we take values of x as –2, -1, 0, 1, 2, 3 and form a table.

Put the values of x on the top line. Put the given equation down the side of the box.

Note You may put the values of x down the side and equation across the top if you want.

x	-2	-1	0	1	2	3
$2x^2$ $-2x$ -3						
y						

Note Each line in the table is different.

Line 1 The $2x^2$ means we put in each value of x square it first and then multiply by 2.

Line 2 The $-2x$ means we put in each value of x and multiply by –2.

Line 3 The -3 goes the whole way along as it has no x.

Note Show the points on graph paper and join the dots freehand.

Using Quadratic Graphs

There are 10 different questions we can be asked. They are asked in different ways so try to know the following questions and how they are solved really well.

Remember when solving questions on this graph that

$$y = f(x)$$

$$f(x) = 2x^2 - 2x - 3$$

$$y = 2x^2 - 2x - 3$$

Note You are allowed an error of ± 0.3

Type 1 Use the graph to find the axis of symmetry.

The axis of symmetry is a vertical line through the lowest (or highest point). The answer to this is always written as $x =$

Type 2 Use the graph to find $f(2.5)$

We have to decide which axis to start on.

Since $y = f(x)$ and the question asks us to find $f(2.5)$ it should be clear that we have the x value. We start at 2.5 on the x axis up (or down) until we hit the curve and then across until we hit the y axis.

Type 3 Use the graph to find the values of x for which $2x^2 - 2x - 5 = 0$.

This is the hardest way that this question can be asked. We must change the question so that it looks like the original question.

$2x^2 - 2x - 5 = 0$ need the - 5 to turn into - 3 so add 2 to the left

$2x^2 - 2x - 5 + 2 = 2$ when we add 2 to the left we add 2 to the right

$2x^2 - 2x - 3 = 2$ can now replace $2x^2 - 2x - 3$ with y.

$y = 2$

3 other ways of asking the same question.

Use the graph to find the values of x for which $2x^2 - 2x - 3 = 2$.

Use the graph to find the values of x for which $f(x) = 2$.

Use the graph to find the values of x for which $y = 2$.

We have to decide which axis to start on.
Since $y = 2$. We start at 2 on the y axis go across both ways until we hit the curve and then down (or up) until we hit the x axis.

Type 4 Use to graph to find the values of x for which $2x^2 - 2x - 3 = 0$.

3 other ways of asking the same question.

Use the graph to find the values of x for which $f(x) = 0$.

Use the graph to find the values of x for which $y = 0$.

Find the roots of the equation.

What they really want us to find are the two points where the graph crosses the x axis.

Type 5 Use to graph to find the values of x for which $2x^2 - 2x - 3 < 0$.

2 other ways of asking the same question.

Use the graph to find the values of x for which $f(x) < 0$.

Use the graph to find the values of x for which $y < 0$.

What the really want us to find this time are the values of x for which the curve is below the x axis.

Type 6 To find the maximum (or minimum) value and point.

The maximum value is the highest value on the graph.

The maximum point is the coordinate of the highest point.

Type 7 To find values of x for which the curve is decreasing.

Decreasing means that as we draw the curve we draw downwards.

Type 8 To find values of x for which $1 + x - 2x^2 > 0$

Must convert the question into the original equation.

$$1 + x - 2x^2 > 0 \quad \text{need the 1 to turn into a 3 so add 2 to both sides}$$
$$2 + 1 + x - 2x^2 > 2$$
$$3 + x - 2x^2 > 2$$
$$y > 2$$

Start on the y axis at the point 2, come across until we hit the curve and come down.
Now we need $y > 2$ which means we need the values of x for which the curve is above the line $y = 2$

Graphs 2

Graph of quadratics and Straight lines.

Type 9 To find values of x for which $f(x) = g(x)$.

What they really want is the values of x for which the curve equals the line.
We need the points of intersection of the curve and line.

Come straight up and down from points where the graphs intersect.

Type 10 To find values of x for which $f(x) < g(x)$.

This time we want the values of x for which the curve is below the line.

Practical Problems

Type 1 A rectangle has one-side x meters long. The rectangle has a total perimeter of 10 meters. Show that the area of the rectangle is $x(5-x)$.

Draw the graph of the function
$f(x)=5x-x^2$ in the domain $0\le x\le 5$, $x\in R$.

To answer the first part draw a diagram of a rectangle, where one side is x

The total perimeter is 10. The 4 sides add up to 10 so that two sides add to 5.

We know one side is x so the other side must be $5-x$.

Area of a rectangle $=$ lenght $\times$ breath

$=x(5-x)$

When we have the diagram drawn the main thing for you to see is that the graph has the same equation as the area of the rectangle. We can replace $f(x)$ for area. Area is put on the y axis and width on the x axis.

With any question on the graph we must decide which axes to start on. We must decide have they given us the area (so that we will start on the vertical axis, go across to hit the graph and the down), or width (so that we start on the horizontal axis, go up to hit the graph and then across).

Type 2 The height of a missile above the ground x seconds after being fired is given by $f(x)$

This time we replace $f(x)$ for height. Height is put on the y axis and time on the x axis.