Complex Numbers Need to Know

Complex Numbers 1

Definition of a complex number

The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

but if $b^2 - 4ac < 0$ it means we have two complex roots as we have the square root of a negative number, which cannot happen.

We say any complex number consists of two parts, a real part and an imaginary part.

Notation

A complex number is usually denoted by z and is said to be of the form $z = a + bi$.

z has two parts the real part called Re(z) which is a and an imaginary part Im(z) which is given by bi.

$$i = \sqrt{-1}$$
$$i^2 = -1$$

To Add or Subtract Complex Numbers

Add real to real and imaginary to imaginary

Multiplication of Complex Numbers

Same method as in algebra . One very important point is that $i^2 = -1$.

Complex Conjugate

If $z = a + bi$ then the conjugate, written $\bar{z} = a - bi$. That is change the sign of the imaginary part.

Division by a Complex Number

Multiply above and below by the complex conjugate of the bottom

Equality of Complex Numbers

If two complex numbers are equal then real equals real and imaginary equals imaginary.

Argand Diagram

Same as x and y axes in coordinate geometry.
The x - axis is now the real axis.
The y - axis is now the imaginary axis.

Note All the material from the line can now be used here.

Complex Numbers 2

Modulus

This is the distance of the complex number from the origin.
If $z = a + bi$ then the modulus of z (written as $|z|$) is given by the formula

$$|z| = \sqrt{a^2 + b^2}$$

Square roots

Step 1 Let whatever is to be square rooted equal $a + bi$.
Step 2 Square both sides.
Step 3 Let real equal real and imaginary equal imaginary.
Step 4 Use simultaneous equations to solve for a and b.

Roots of a Polynomial

If z_1 is a root of $az^3 + bz^2 + cz + d = 0$ where a, b, c and d are real then $\overline{z}_1$ is also a root.

This means that as long as the coefficient of the polynomial are real then the roots occur in conjugate pairs.

Quadratic Equations

There are 4 types of question that appear here and we will try to do one of each.

Type 1 To find the roots of a quadratic:

The roots of the quadratic equation $ax^2 + bx + c = 0$ are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

even if a, b or c are real or unreal.

Type 2 To form a quadratic equation given the roots.

Use $z^2 - (\alpha + \beta)z + \alpha\beta = 0$

Type 3 To find a root of a quadratic given one of the roots.

Type 4 To find unknown coefficient of quadratic given the roots.

Complex Numbers 3

The material in this class is harder than the first class so a lot more effort is required here.

This class will show you how to solve cubic equations and how to write a complex number in polar from.

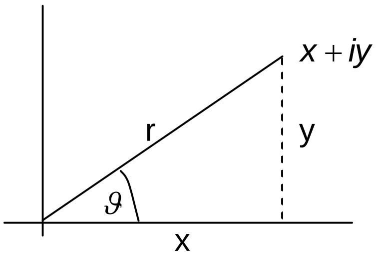
To solve Cubic Equations

Polar Co-ordinates

The complex number $z = x + iy$ can most easily be changed to the polar form by means of an argand diagram.

$r = |z|$

$\vartheta =$ argument $=$ angle made with the line and the positive x - axis.



From the diagram it will be easy to find the values for r and ϑ

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan\vartheta = \frac{y}{x}$$

$$z = x + iy$$

$$= r(\cos\vartheta + i\sin\vartheta)$$

This could be written in what is called general polar form is

$$z = r(\cos(\vartheta + 2n\pi) + i\sin(\vartheta + 2n\pi))$$

Properties of complex numbers in polar form

When $z_1 = r_1(\cos\vartheta_1 + i\sin\vartheta_1)$ and $z_2 = r_2(\cos\vartheta_2 + i\sin\vartheta_2)$

then

$$z_1 z_2 = r_1 r_2(\cos(\vartheta_1 + \vartheta_2) + i\sin(\vartheta_1 + \vartheta_2))$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\vartheta_1 - \vartheta_2) + i\sin(\vartheta_1 - \vartheta_2))$$

$$\frac{1}{z_1} = \frac{1}{r_1}(\cos\vartheta_1 - \sin\vartheta_1)$$

Note Since the question said “leave your answer in the form $a + bi$ ” you must write the answer as $0 + 40i$ or you will lose 3 marks.

Complex Numbers 4

De Moivre's Theorem

If $z = r(\cos\vartheta + i\sin\vartheta)$

$$z^n = r^n(\cos n\vartheta + i\sin n\vartheta)$$

To raise a complex number to a power

we keep on subtracting 360^o until we find an angle between 0^o and 360^o .

To prove trigonometric equations

Prove $\sin 4\theta = 4\sin\theta(2\cos^3\theta - \cos\theta)$

Must start with $(\cos\theta + i\sin\theta)^4$ and expand in two different ways using De Moivre’s Theorem and the Binomial Theorem.

To find the roots of a complex equation.

When we have a fraction in the power we must use general polar form.

Trigonometry Identity

Prove $\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$

Start with $z = \cos\theta + i\sin\theta$ and $\frac{1}{z} = \cos\theta - i\sin\theta$

$$z + \frac{1}{z} = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta = 2\cos\theta$$

$$z - \frac{1}{z} = \cos\theta + i\sin\theta - (\cos\theta - i\sin\theta) = 2i\sin\theta$$