Matrices Need to Know

Terminology

Matrices are elements arranged in rows and columns.

In general the matrix which has m rows and n columns is called an m by n matrix.

A square matrix is one for which the number of rows is equal to the number of columns.

$0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is called the zero matrix.

A diagonal matrix is one of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

Basic Operations

Just like any number system we have to deal with basic operations.

Addition and Subtraction

If they have the same order then add and subtract corresponding terms.

Note $A + B = B + A$

$(A + B) + C = A + (B + C)$

Multiplication by a scalar

Multiply each element by the scalar $k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$

Multiplication of matrices

A and B can be multiplied only if the number of columns of A equal the number of rows of B.

If A is an m by n matrix (i.e. m rows and n columns) and B is a p by q matrix, then the product AB exists only if n $=$ p and the result is an m by q matrix.

Note AB $\neq$ BA

Identity Matrix for Multiplication

The matrix $I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ is the identity matrix for multiplication so $AI=IA=A$

Note $A^2=A\times A$ no short cuts.

Inverse of a 2 by 2 Matrix

The inverse of the matrix $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$ is given by $A^{-1}=\dfrac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$

The expression $ad-bc$ is called the determinant (denoted by Δ) of the matrix $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

If $\Delta=0$ then $\dfrac{1}{\Delta}$ is undefined and the matrix has no inverse and is said to be singular.

Using the inverse to solve matrix equations

If $AX = Y$ to find the matrix X pre multiply both sides by A^{-1} because

$$AX = Y$$
$$X = A^{-1}Y$$

If $AX = Y$ to find the matrix A post multiply both sides by X^{-1} because

$$AX = Y$$
$$A = YX^{-1}$$

Matrices to solve Simultaneous Equations

Diagonal Matrices

Property 1 $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^n = \begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$

Property 2 If P is an invertible matrix and M is some other matrix then $(P^{-1}MP)^n = P^{-1}M^nP$.

Both these are proven using proof by induction.

Find $P^{-1}NP$ and hence find N^{20}.

Note Order of multiplication is vital.