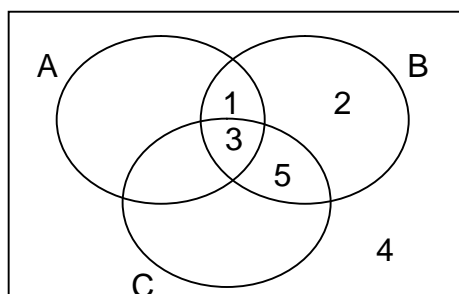


Sets 2

This class has the harder stuff of three set problems.

Three sets problems



A Venn diagram with 3 sets has extra regions for us to explain.

Region 1 = $(A \cap B) \setminus C$ in both A and B but not C.

Region 2 = $B \setminus (A \cup C)$ in B only.

Region 3 = $A \cap B \cap C$ in all 3.

Region 4 = $(A \cup B \cup C)'$ outside all 3 sets.

Regions 1 and 3 = $A \cap B$

Regions 1, 2, 3 and 5 = B

Example 1 U is the universal set and A, B and C are three subsets of U.

$$B = \{a, b, c, d\}$$

$$(B \cap C) \setminus A = \{c\}$$

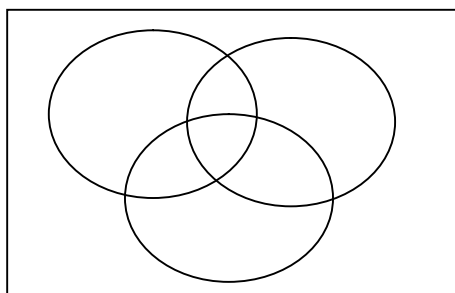
$$A \cap B = \{a, b, d\}$$

$$A \setminus C = \{d, e, f\}$$

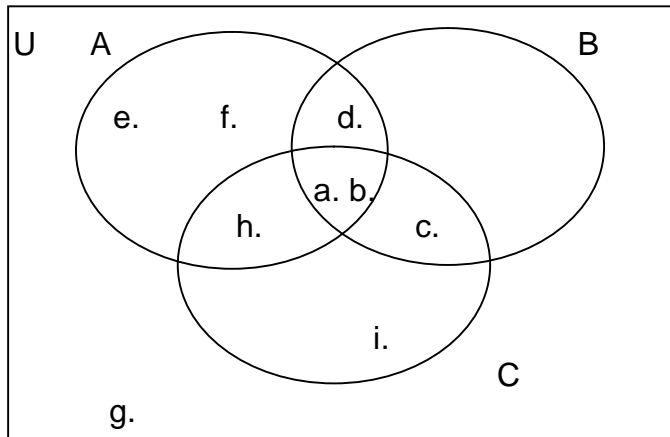
$$(A \cup B \cup C)' = \{g\}$$

$$C \setminus B = \{h, i\}$$

$$A' = \{c, g, i\}$$



Copy the Venn diagram and complete it to show the elements in each part of each set.



Fill in the set with the smallest number of elements first.

$$(B \cap C) \setminus A = \{c\} \text{ and } (A \cup B \cup C)' = \{g\}$$

$C \setminus B = \{h, i\}$ could be in $A \cap C$ or C only but given $A' = \{c, g, i\}$ then this means that h must be in the set A and i must be outside in the set C only.

$A' = \{c, g, i\}$ all of which are already done.

$A \cap B = \{a, b, d\}$ could be in two possible regions but since $A \setminus C = \{d, e, f\}$ then a and b are in the intersection of all 3 and d is in $(A \cap B) \setminus C$.

$A \setminus C = \{d, e, f\}$ means we put e and f in A only as d is already accounted for.

$B = \{a, b, c, d\}$ all of which are accounted for.

Example 2 A survey was taken of a group of 44 students, each of whom was studying one or more of three subjects History, Geography and Art.

28 students studied History.

30 students studied Geography.

22 students studied Art.

6 students studied History only.

15 students studied both History and Geography.

3 students studied all three subjects.

- (i) Use a Venn diagram to find the number of students who studied History and Geography but not Art.
- (ii) How many students studied History and Art but not Geography?

(iii) Find the number of students who studied Geography only.

We have to fill the Venn diagram starting with the inside and work out.

3 students studied all three subjects so put 3 in the centre.

15 students studied both History and Geography but 3 have being accounted for (do all three) so we put 12 in H and G but not A.

6 students studied History only. At this stage we have 3 of the four regions of History filled so we can fill in the rest.

28 students studied History but 21 (6+12+3) have being accounted for so we put the 7 that are left into History and Art but not Geography.

Since we do not know how many are in Art and Geography we let this region = x .

We need to know how many do Geography only (in terms of x)

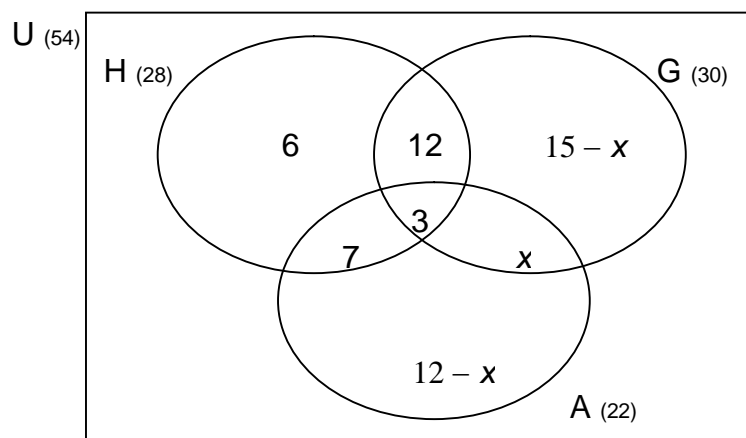
In Geo there are 30 in total of which we have accounted for 9 and 3 and x .

$$\begin{aligned}\text{Geo only} &= 30 - (12 + 3 + x) \\ &= 30 - 1(15 + x) \\ &= 30 - 15 - x \\ &= 15 - x\end{aligned}$$

Note Put in 1 in-front of the bracket to try to stop sign mistakes.

Now we need to figure out Art only.

$$\begin{aligned}&22 - (7 + 3 + x) \\ &= 22 - 1(10 + x) \\ &= 22 - 10 - x \\ &= 12 - x\end{aligned}$$



Add up all the regions to find the value of x

$$6 + 12 + 15 - x + 7 + 3 + x + 12 - x = 44$$

$$55 - x = 44$$

$$-x = -11$$

$$x = 11$$

- (i) History and Geography but not Art. Answer = 12
- (ii) History and Art but not Geography. Answer = 7
- (iii) Geography only. Answer $15 - 11 = 4$

Example 3 A survey was taken of a group of 54 students, each of whom was studying one or more of three subjects A, B or C.

6 students studied B and C.

5 students studied A and C.

3 times as many students studied A and B as studied all 3 subjects.

20 students altogether studied B.

17 students studied C only.

14 students studied A only.

Using x to represent those students who studied all 3 subjects, illustrate the above information in a Venn diagram and hence calculate the value of x .

x students studied all three subjects so put x in the centre.

14 students studied A only means that there are 14 students in the A only region as shown.

17 students studied C only means that there are 17 students in the C only region as shown.

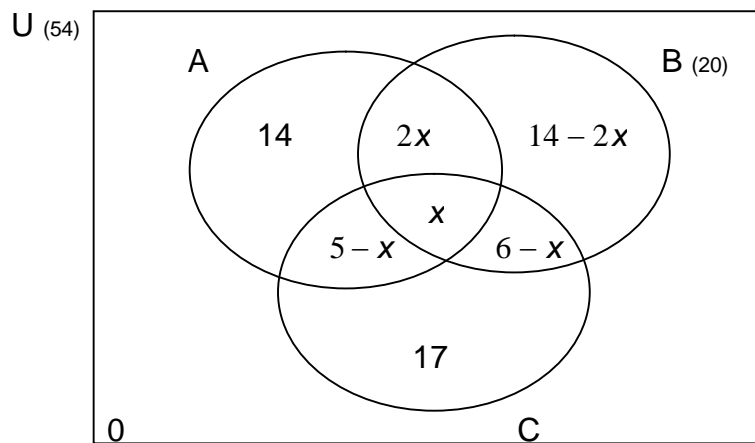
6 students studied B and C but x have being accounted for (do all three) so we put $6 - x$ in B and C but not A.

5 students studied A and C but x have being accounted for (do all three) so we put $5 - x$ in A and C but not B.

3 times as many students studied A and B as studied all 3 subjects so that means there are $3x$ people in A and B but x have being accounted for (do all three) so we put $3x - x = 2x$ in A and B but not C.

20 students altogether studied B but 3 of the regions ($2x$, x and $6 - x$) are accounted for.

$$\begin{aligned}
 \text{B only} &= 20 - (2x + x + 6 - x) \\
 &= 20 - 1(6 + 2x) \\
 &= 20 - 6 - 2x \\
 &= 14 - 2x
 \end{aligned}$$



$$\begin{aligned}
 14 + 2x + 14 - 2x + 5 - x + x + 6 - x + 17 &= 54 \\
 56 - x &= 54 \\
 -x &= -2 \\
 x &= 2
 \end{aligned}$$