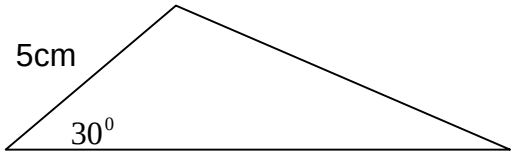
Trigonometry 6

In this class we will deal with the methods for solving non right angled triangles.

Non right-angled triangles

Example 1 Find the area of the triangle shown:



12cm

Area of a triangle $= \frac{1}{2}ab\sin C$, which means we multiply one length by another by sin of the angle between the two lengths.

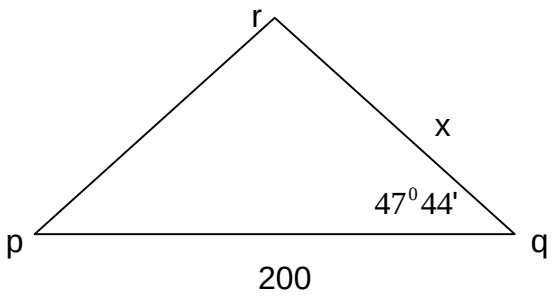
$$\text{Area} = \frac{1}{2}abSinC$$

$$= \frac{1}{2}(5)(12)\sin 30$$

$$= (0.5)(5)(12)(0.5)$$

$$= 15$$

Example 2 The area of the triangle pqr is $9028\,m^2$, |pq| = 200m and $|\angle pqr| = 47^0 44'$. Find |qr|.



$$\text{Area} = \frac{1}{2}abSinC$$

$$9028 = \frac{1}{2}(200)(x)\sin 47^0 44'$$

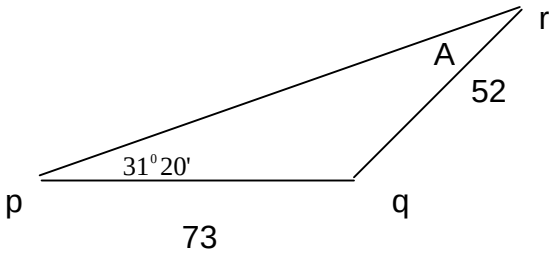
$$9028 = (0.5)(200)(x)(0.74)$$

$$9028 = 74x$$

$$74x = 9028$$

$$x = 122\text{m}$$

Example 3 Two lighthouses, p and q, are 73 km apart. q is directly East of p. Another lighthouse, r, is situated 52 km from q. The bearing of r from p is $E\,31^0 20'\,N$. Calculate |pr|, correct to the nearest kilometre.



Is the triangle right angled? No so must move on.

Have we an angle and it's opposite length so that we can use the sin rule? Yes.

The next question is what can we find? Well we have the length |pq| and it's opposite angle and we have the length |qr| so we can find it's opposite angle which is marked in the diagram as the angle A.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{73} = \frac{\sin 31^0 20'}{52}$$

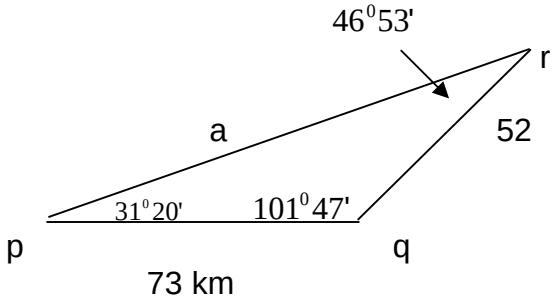
$$52\sin A = 73\sin 31^0 20'$$

$$52\sin A = 73(0.52)$$

$$\sin A = 0.73$$

$$A = 46.88^0$$

$$A = 46^0 53'$$



Add $31^0 20' + 46^0 53' = 78^0 13'$ now we can find the third angle.
$180^0 - 78^0 13' = 101^0 47'$

Use the sin rule again to find |pr| which we will call a.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 101^0 47} = \frac{52}{\sin 31^0 20'}$$

$$\frac{a}{0.9789} = \frac{52}{0.5348}$$

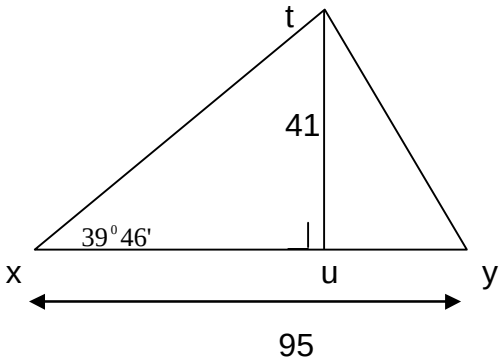
$$0.5348a = 52(0.9789)$$

$$0.5348a = 50.9$$

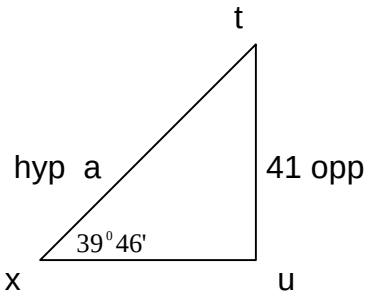
$$a = \frac{50.9}{0.5348} = 95.1 = 95\text{km} \text{ to the nearest km}$$

Example 4 t, x, u and y are points on level ground, x, u and y in a straight line.

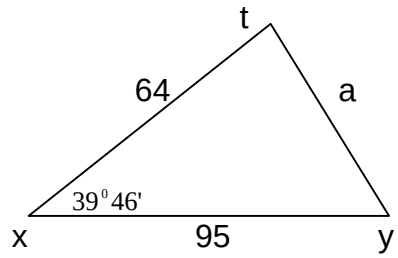
From x the direction of t is east $39^0 46'$ North.
u is directly South of t.
|xy| = 95m and |tu| = 41m
Find |ty|, correct to the nearest metre.



There are 3 possible triangles for us to draw out but the question is which one has got a length in it.



$$\sin 39^0 46' = \frac{41}{a}$$
$$0.6396a = 41$$
$$a = 64.1$$
$$= 64$$



Is the triangle right angled? No so must move on.

Have we an angle and it's opposite length, can we use the sin rule? No so move on. The only choice we are left with is to use the cosine rule.

Mark in missing side |ty| as a

.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 64^2 + 95^2 - 2(64)(95)\cos 39^0 46'$$

$$= 4096 + 9025 - 9346.85$$

$$= 3774.14$$

$$a = 61.4$$

$$a = 61\text{m}$$