

Trigonometry

Class 1

Example 1 Find $\cos 54^{\circ} 29'$ correct to 2 decimal places.

Example 2 If $\tan A = 1.23$ find a value for A .

Example 3 Convert 60° in terms of π .

Example 4 Convert $\frac{\pi}{6}$ to degrees

Example 5 Convert 2.5 rads to degrees

Example 6 Express in surd form $\cos 135^{\circ}$

Example 7 Find in surd form

(i) $\tan 330^{\circ}$

(ii) $\sin 240^{\circ}$

Class 2

Example 1 Solve $\cos A = -\frac{\sqrt{3}}{2}$ where $0 \leq A \leq 360^{\circ}$.

Example 2 Solve the equation $\tan A = -1$ where $0^{\circ} \leq A \leq 360^{\circ}$.

Example 3 Solve the equation $\cos \theta = -0.45$, where $0 \leq A \leq 2\pi$.

Example 4 Find $\sin(-210)^{\circ}$ in the form $\frac{a}{b}$

Example 5 Express $\sin 75$ in surd form

Example 6 If $\cos A = \frac{3}{5}$ and $\sin B = \frac{12}{13}$ find

- (i) $\cos(A + B)$
- (ii) $\cos(45 - A - B)$

Example 7 If $\tan(A + B) = 3$ and $\tan A = 2$ find $\tan B$

Class 3

Example 1 If $\sin A = \frac{3}{5}$ find the value of $\sin 2A$ without tables or calculator.

Example 2 If $\sin 2A = \frac{3}{5}$ find the value of $\sin A$ without tables or calculator.

Example 3 Prove $\cos 3A = 4 \cos^3 A - 3 \cos A$

Example 4 Find in surd form $\tan 22 \frac{1}{2}^\circ$

Example 5 Prove $\frac{\sin A}{1 + \cos A} = \tan \frac{A}{2}$

Class 4

Example 1 Express $\cos 3A \sin A$ as a sum or difference.

Example 2 Express each of the following as a product

(i) $\cos 3A - \cos A$

(ii) $\sin 5A - \sin 2A$

Example 3 Solve the equation $\tan A = -\sqrt{3}$ where $0^\circ \leq A \leq 360^\circ$

Example 4 Solve $\cos A = 0$, where $0 \leq A \leq 2\pi$.

Example 5 Solve $\cos 2A = \frac{\sqrt{3}}{2}$, where $0 \leq A \leq 2\pi$.

Example 6 Solve the equation $\sin A = \sqrt{3} \cos A$, where $0 \leq A \leq 2\pi$.

Example 7 Solve $2\sin^2 x - 3\sin x + 1 = 0$

Example 8 Solve $\cos^2 A - 2\sin A - 1 = 0$, where $0^\circ \leq A \leq 360^\circ$.

Example 9 Solve the equation $\cos 3x = \cos x$ for $0 \leq x \leq 180^\circ$

Example 10 Solve the equation $\sin 3x + \sin 2x + \sin x = 0$ for $0 \leq x \leq 180^\circ$

Example 11 Solve $\cos 2\theta + \cos \theta + 1 = 0$ for $0 \leq \theta \leq 2\pi$

Example 12 Solve $\sin \theta + \sqrt{3} \sin \frac{\theta}{2} = 0$ for $0 \leq \theta \leq 2\pi$

Example 13 Solve $\cos A + \sin A + 1 = 0$ where $0^\circ \leq A \leq 360^\circ$.

Class 5

Example 1 A ladder, which is 6m long, leans against a vertical wall. The foot of the ladder is on level ground at a distance of 1m from the bottom of the wall. Find the measure of the angle that the ladder makes with the ground to the nearest degree.

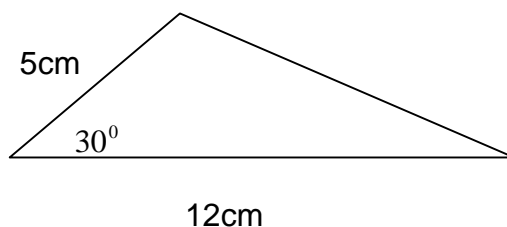
Example 2 A man stands on top of a vertical cliff. He spots a buoy 27 m from the base of the cliff at an angle of depression of 35° . How high is the cliff to two decimal places?

Example 3 If $\sin A = \frac{5}{13}$ and A is an acute angle write down the value of $\cos A$ without tables or calculator.

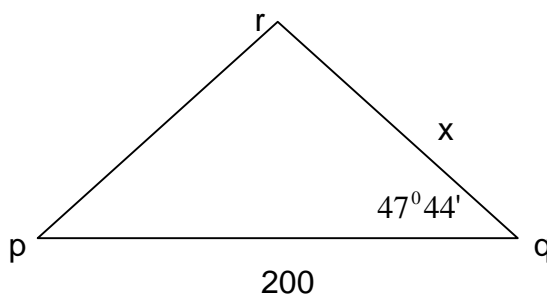
Example 4 Find the length of the sides of a square that has a diagonal which is 10cm long.

Class 6

Example 1 Find the area of the triangle shown:



Example 2 The area of the triangle pqr is 9028 m^2 , $|pq| = 200\text{m}$ and $|\text{pqr}| = 47^\circ 44'$. Find $|qr|$.



Example 3 Two lighthouses, p and q, are 73 km apart. Q is directly East of p. Another lighthouse, r, is situated 52 km from q. The bearing of r from p is E 31°20' N. Calculate |pr|, correct to the nearest kilometre.

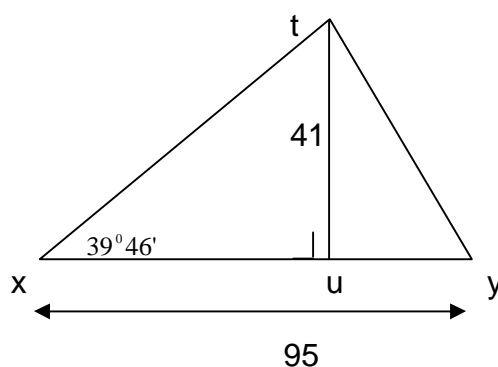
Example 4 t, x, u and y are points on level ground, x, u and y in a straight line.

From x the direction of t is east 39°46' North.

u is directly South of t.

|xy| = 95m and |tu| = 41m

Find |ty|, correct to the nearest metre.



Class 7

Example 1 Find the height of a building if the angle of elevation at a point on its roof changes from 30° to 60° on walking 14m towards it.

Example 2 A vertical flagpole stands on horizontal ground. The angle of elevation of the top of the pole from a certain point on the ground is θ . From a point 10 meters closer to the pole the angle of elevation is β . Show that the height of the pole is

$$\frac{10 \sin \theta \sin \beta}{\sin(\beta - \theta)}$$

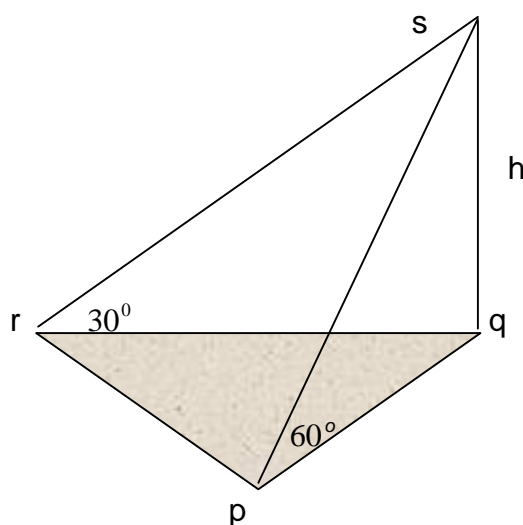
Example 3 Find the largest angle of the triangle whose sides are in the ratio 4 : 5 : 6.

Example 4 A parallelogram has sides of 40 cm and 28 cm. One diagonal is 50 cm long. How long is the other diagonal to one decimal place.

Example 5 If the length of the arc pq is 6cm and the radius of the sector is 5cm find the shaded region to one decimal place.

Example 6 p , q and r are three points on level ground. $[sr]$ is a vertical tower of height h . The angles of elevation of the top of the tower from p and q are 60° and 30° respectively.

- (i) Express $|pr|$ and $|qr|$ in terms of h .
- (ii) Find $|qp|$ in terms of h , if $\tan \angle qrp = \sqrt{8}$



Class 8

Example 1 Prove each of the following identities

$$(i) \quad \frac{\sin A}{\sqrt{1 - \sin^2 A}} = \tan A$$

$$(ii) \quad \frac{\sin A}{\operatorname{cosec} A} + \frac{\cos A}{\sec A} = 1$$

Example 2 Prove each of the following identities

$$(i) \quad \frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} = 2 \operatorname{cosec}^2 A$$

$$(ii) \quad \frac{\tan A + \sin A}{1 + \sec A} = \sin A$$

Example 3 Prove the following

$$(i) \quad \frac{\cos 2A}{\cos A + \sin A} = \cos A - \sin A$$

$$(ii) \quad \frac{\sin 2A}{1 + \cos 2A} = \tan A$$

Example 4 Prove for any triangle

$$(i) \quad \frac{a}{b} = \frac{\sin A}{\sin B}$$

$$(ii) \quad a^2 = abc \cos C + acc \cos B$$

Class 9

Example 1 Find $\cos\left(2 \sin^{-1} \frac{3}{5}\right)$

Example 2 Express $\sin(\cos^{-1} x)$ in terms of x .

Example 3 Draw $y = \sin A$ in the domain $0 \leq A \leq 2\pi$

Example 4 Draw $y = \cos A$ in the domain $0 \leq A \leq 2\pi$

Example 5 Draw $y = \cos A$ in the domain $0 \leq A \leq 2\pi$

Proofs Class

Proof 1 Prove the cosine rule

Proof 2 To prove $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Proof 3 To prove $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Proof 4 To prove $\sin(A + B) = \sin A \cos B + \cos A \sin B$

Proof 5 To prove $\sin(A - B) = \sin A \cos B - \cos A \sin B$

Proof 6 To Prove $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Proof 7 To Prove $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Proof 8 To prove $\cos 2A = \cos^2 A - \sin^2 A$

Proof 9 To prove $\sin 2A = 2 \sin A \cos A$

Proof 10 To prove $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Proof 11 To prove $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

Proof 12 To prove $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

Proof 13 Prove $\cos 3A = 4 \cos^3 A - 3 \cos A$