

# Sets 1

This class contains basic notation, two set problems and maximum/ minimum problems.

## Notation

A set is defined as a collection of well defined objects and must be contained inside brackets  $\{ \}$ .

**Note** Sets are named using capital letters.

$\in$  is element – part of a set.

$\notin$  is not an element.

$\cup$  is union – everything in two or more sets.

$\cap$  is intersection – common to both sets.

$\subset$  is subset – a set within a set.

$\not\subset$  is not a subset.

$\emptyset = \{ \}$  is the null set – a set with nothing in it.

$A'$  is A complement – everything outside the set A.

$A \setminus B$  is A less B – what is in the set A but not including what's in the set B (A only).

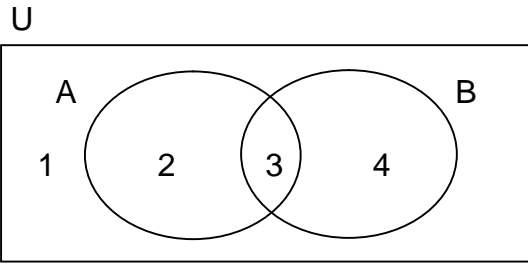
$\#A$  is cardinal number of A – the number of elements in the set A.

**Example 1** Write out all the subsets of  $A = \{a, b, c\}$

Answers  $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \emptyset, \{a, b, c\}$

**Note** Every set has two improper subsets – the null set and the set itself.

## Two Set Problems



In the above venn diagram let 1, 2, 3, 4 stand for the name of each region.

Region 1 =  $(A \cup B)' = U \setminus (A \cup B)$  – outside both sets or neither set.

Region 2 =  $A \setminus B$  which is A only.

Region 3 =  $A \cap B$  which is common to both sets.

Region 4 =  $B \setminus A$  which is B only.

Regions 2+3 = A

Regions 3+4 = B

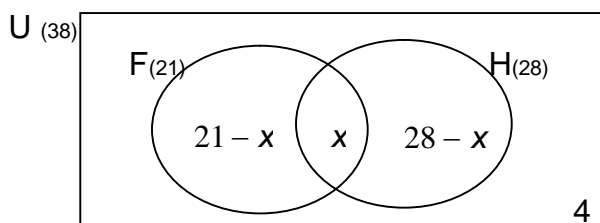
Regions 2+3+4 =  $A \cup B$

**Example 2** 38 people in a group. 21 play football and 28 play hurling. 4 play no sport, how many play both?

When we do not know the number of people that do both we put  $x$  in the middle and then subtract out.

There are 21 people that play football but of those  $x$  play both then the number of people that play football only must be  $21 - x$ .

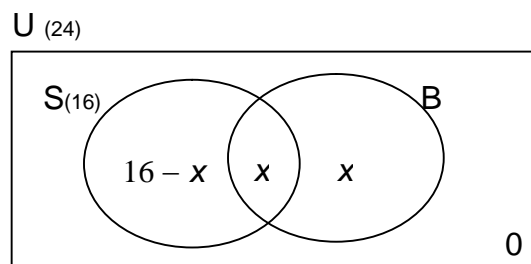
There are 28 people that play hurling but of those  $x$  play both then the number of people that play hurling only must be  $28 - x$ .



We know the number of elements of each set and we know the number of people in total so the when we add up all the regions they should add up to 38.

$$\begin{aligned} 21 - x + x + 28 - x &= 38 \\ 49 - x &= 38 \\ -x &= -11 \\ x &= 11 \end{aligned}$$

**Example 3** There are 24 students in a class. 16 study science. Twice as many study business as study both, if everyone studies at least one subject how many study business only?



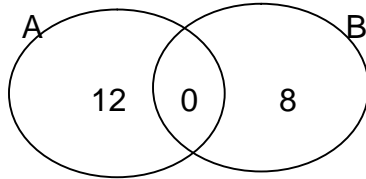
$$\begin{aligned} 16 - x + x + x &= 24 \\ 16 + x &= 24 \\ x &= 8 \end{aligned}$$

**Note** The set B is hard to figure out so that you have to read the question over and over. Twice as many study business as study both. There are  $x$  in both so there must be  $2x$  in total in the set B (twice as many). Since there are  $2x$  in the set B and of these  $x$  are in both then there must be  $2x - x = x$  in the region B only.

## Max and Min problems

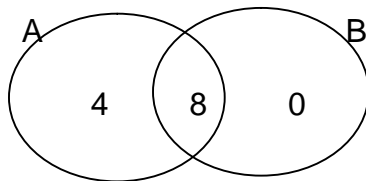
**Example 4** If  $\#A = 12$  and  $\#B = 8$  find  
 the maximum value of  $A \cup B$ .  
 the minimum value of  $A \cup B$ .

To find the maximum value of  $A \cup B$  put the smallest possible number of elements in the intersection, which in this case is 0. That means all 12 elements of A are in A only and of B are in B only



Maximum value of  $A \cup B = 12 + 8 = 20$

To find the minimum value of  $A \cup B$  put the largest possible number of elements in the intersection, which in this case is 8 since this is the smallest number of elements in the two sets.

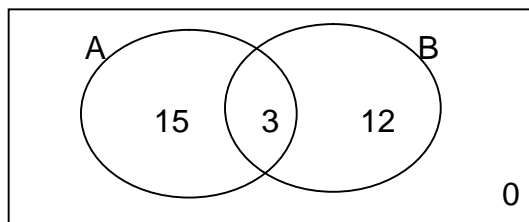


Minimum value of  $A \cup B = 4 + 8 = 12$

**Example 5** In a group of 30 people, 18 read paper A and 15 read paper B find  
 the maximum number of people that read a paper  
 the minimum number of people that read a paper.

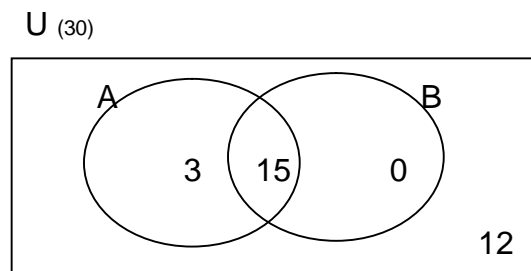
This question is different in that we have a Universal set so that there is a cap on the number of people. When we want to figure out the smallest possible number that could have done both we have to find the overlap. Smallest possible number that did both is  $18 + 15 - 30 = 3$

$U_{(30)}$



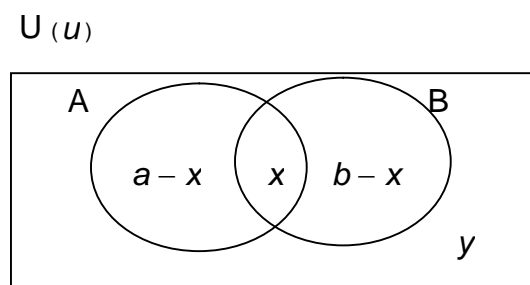
The maximum number of people that read a paper  $= 15 + 3 + 12 = 30$

To find the minimum value of  $A \cup B$  put the largest possible number of elements in the intersection, which in this case is 15 since this is the smallest number of elements in the two sets.



The minimum number of people that read a paper =  $3 + 15 = 18$

**Example 6** When  $\#U = u$ ,  $\#A = a$ ,  $\#B = b$ ,  $\#(A \cap B) = x$  and  $\#(A \cup B) = y$   
Write  $y$  in terms of  $a$ ,  $b$ ,  $x$  and  $u$ .



$$a - x + x + b - x + y = u$$

$$a + b - x + y = u$$

$$y = u - a - b + x \text{ leave } y \text{ on the left}$$