Trigonometry 8

This class contains identities.

Trigonometric Identities

Use the maths tables to the full and to work with each side separately.
Always keep an eye on what you are trying to find out.

There is also a lot of algebra that we may need here like

(i) The square of two terms.

(ii) The difference of two squares.

(iii) How to deal with complex fractions in terms of sin and cos.

(iv) How to add and subtract fractions.

One identity that may arise a lot here is

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A = 1 - \cos^2 A$$

$$\cos^2 A = 1 - \sin^2 A$$

Note Only change a term if we gain by the change.

Write everything in terms of sin and cos. If we cannot use the tables use algebra. You should be able to see where it is going after 5 lines or so.

Example 1 Prove each of the following identities

(i) $$\frac{\sin A}{\sqrt{1-\sin^2 A}} = \tan A$$

(ii) $$\frac{\sin A}{\operatorname{cosec} A} + \frac{\cos A}{\sec A} = 1$$

(i) $\frac{\sin A}{\sqrt{1-\sin^2 A}} = \tan A$

$$\frac{\sin A}{\sqrt{\cos^2 A}} = \frac{\sin A}{\cos A}$$

$$\frac{\sin A}{\cos A} = \frac{\sin A}{\cos A}$$

(ii) $\frac{\sin A}{\cos ecA} + \frac{\cos A}{\sec A} = 1$

$$\frac{\frac{\sin A}{1}}{\frac{1}{\sin A}} + \frac{\frac{\cos A}{1}}{\frac{1}{\cos A}} = 1$$

$$\sin^2 A + \cos^2 A = 1$$

Example 2 Prove each of the following identities

(i) $\frac{1}{1+\cos A} + \frac{1}{1-\cos A} = 2\cos ec^2 A$

(ii) $\frac{\tan A + \sin A}{1+\sec A} = \sin A$

Need to find a common denominator on the left.

(i) $\frac{1}{1+\cos A} + \frac{1}{1-\cos A} = 2\cos ec^2 A$

$$\frac{1-\cos A + 1 + \cos A}{(1+\cos A)(1-\cos A)} = \frac{2}{\sin^2 A}$$

$$\frac{2}{1-\cos^2 A} = \frac{2}{\sin^2 A}$$

In the example that follows change everything to cos and sin for the simple reason that we cannot do anything else. Then use complex fractions and the tables to prove the identity

(ii) $$\frac{\tan A + \sin A}{1 + \sec A} = \sin A$$

$$\frac{\frac{\sin A}{\cos A} + \frac{\sin A}{1}}{1 + \frac{1}{\cos A}} = \sin A$$

$$\frac{\frac{\sin A + \sin A \cos A}{\cos A}}{\frac{\cos A + 1}{\cos A}} = \sin A$$

$$\frac{\sin A + \sin A \cos A}{\cos A + 1} = \sin A$$

$$\frac{\sin A(1 + \cos A)}{\cos A + 1} = \sin A$$

$$\sin A = \sin A$$

Double angles

Example 3 Prove the following

(i) $$\frac{\cos 2A}{\cos A + \sin A} = \cos A - \sin A$$

(ii) $$\frac{\sin 2A}{1 + \cos 2A} = \tan A$$

(i) $$\frac{\cos 2A}{\cos A + \sin A} = \cos A - \sin A$$

$$\frac{\cos^2 A - \sin^2 A}{\cos A + \sin A} = \cos A - \sin A$$

$$\frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A + \sin A} = \cos A - \sin A$$

$$\cos A - \sin A = \cos A - \sin A$$

(ii) $$\frac{\sin 2A}{1 + \cos 2A} = \tan A$$

Note There is always a hint in the question. Do you see on the left we have the angle 2A but on the right we just have the angle A. What does this mean? Well the angle has being halved in some way. We need to use the maths tables.

In the tables

$$\sin 2A = \frac{2\tan A}{1 + \tan^2 A} \quad \text{and} \quad \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\frac{\sin 2A}{1 + \cos 2A} = \tan A$$

$$\frac{\dfrac{2\tan A}{1 + \tan^2 A}}{1 + \dfrac{1 - \tan^2 A}{1 + \tan^2 A}} = \tan A$$

$$\frac{\dfrac{2\tan A}{1 + \tan^2 A}}{\dfrac{1 + \tan^2 A + 1 - \tan^2 A}{1 + \tan^2 A}} = \tan A$$

$$\frac{2\tan A}{2} = \tan A$$

Example 4 Prove for any triangle

(i) $\frac{a}{b} = \frac{\sin A}{\sin B}$

(ii) $a^2 = ab\cos C + ac\cos B$

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Note Once you see small letters in identities then we must use either the sin rule or cosine rule or maybe even both rules.

(i) $\frac{a}{b} = \frac{\sin A}{\sin B}$

$$a\sin B = b\sin A$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

(ii) $a^2 = ab\cos C + ac\cos B$

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 - a^2 - b^2 = -2ab\cos C$$

$$a^2 + b^2 - c^2 = 2ab\cos C$$

$$\frac{a^2 + b^2 - c^2}{2} = ab\cos C$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 - a^2 - c^2 = -2ac\cos B$$

$$a^2 + c^2 - b^2 = 2ac\cos B$$

$$\frac{a^2 + c^2 - b^2}{2} = ac\cos B$$

$$a^2 = ab\cos C + ac\cos B$$

$$a^2 = \frac{a^2 + b^2 - c^2}{2} + \frac{a^2 + c^2 - b^2}{2}$$

$$a^2 = \frac{a^2 + b^2 - c^2 + a^2 + c^2 - b^2}{2}$$

$$a^2 = a^2$$