Sequence and Series

Class 1

Example 1 The general term of a sequence is given by $U_n = 3n - 4$

Find (a) U_1
(b) U_6
(c) $U_{n+1} - U_n$
(d) For what value of n is $U_n = 32$

Example 2 Write out the first 4 terms of the series given by $\sum_{n=1}^{n} n^3$

Example 3 If $S_n = n^2 - 6n$ find U_n in terms of n.

Example 4 Prove that the sequence $T_n = 4n + 3$ is arithmetic, find the values of a and d.

Example 5 The first three terms of an arithmetic sequence are 5, 8, 11 find T_n the n^{th} term and hence find T_{12}.

Example 6 The first term of arithmetic sequence is 7 and the common difference is –2. Which term of the sequence is –351?

Example 7 The sixth term of arithmetic sequence is 13 and the tenth term is 5 find the first term and the common difference.

Class 2

Example 1 Find the sum of the first 12 terms of 6 + 2 – 2 – 6 - - - . Find the sum of the next 12 terms.

Example 2 The first term of an arithmetic sequence is -4 and the common difference is 2.

How many terms of the series must be added to give a sum of 126?

Example 3 The first three terms in an arithmetic sequence are $x-3$, $2x+7$ and $x+5$. Find the value of x.

Example 4 Find three consecutive terms of an arithmetic sequence which add to 12 and multiply to 48.

Example 5 If S_n of an arithmetic series is given by $S_n = n^2 - 3n$ find a and d.

Example 6 p, q and r are three numbers in an arithmetic sequence. Prove that $p^2 + r^2 \geq 2q^2$.

Class 3

Example 1 Prove that the sequence $T_n = 3^n$ is geometric.

Example 2 The first three terms of a geometric sequence is 4, -12, 36 find a the first term and r the common ratio.

Example 3 The nth term of a geometric series is given by $T_n = 5(2)^n$

Example 4 The first three terms of a geometric sequence are 5, -15, 45 find T_n the nth term and hence find T_8.

Example 5 The fourth term of a geometric sequence is 36 and the sixth term is 144 find the first term and the common ratio r where $r > 0$.

Example 6 The first three terms of a geometric sequence are $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, ---$

Write down the value of a and the value of r.
Find S_6, the sum of the first 6 terms.

Example 7 The first three terms of a geometric series are $2+6+18+---$
Find a and r.
How many terms of the series must be added to give a total of 6,560.

Class 4

Example 1 Find the values of x for which 2, $x+1$, 32 are the first three terms in a geometric sequence.

Example 2 Find three consecutive terms of a geometric sequence, which add to 14 and multiply to 64.

Example 3 $a, b, c, d,$ are the first, second, third and fourth terms of a geometric sequence, respectively.
Prove that $a^2 - b^2 - c^2 + d^2 \geq 0$

Example 4 Find S_∞ of the geometric series.

(i) $1+\frac{1}{2}+\frac{1}{4}+---$

(ii) $\sum_{n=1}^{\infty}\left(\frac{x}{x+2}\right)^n$ where $x>1$

Example 5 Express $0.\dot{5}$ in the form $\frac{p}{q}$ where $p, q \in Z, q \neq 0$

Example 6 Express $1.5\dot{4}\dot{7} = 1.547474747...$ in the form $\frac{p}{q}$ where $p, q \in Z, q \neq 0$.

Class 5

Example 1 Write out the first 5 terms in the sequences

(a) $U_1 = 2$, $U_{n+1} = U_n + 3$

(b) $U_0 = 1$, $U_1 = 1$, $U_{n+2} = U_{n+1} + U_n$

Example 2 Given $U_n = \dfrac{2n+3}{n+5}$ determine whether the sequence is increasing or decreasing.

Example 3 Find S_n and hence S_∞ of the series

$$\frac{1}{2(4)} + \frac{1}{3(5)} + \frac{1}{4(6)} + - - - + \frac{1}{(n+1)(n+3)}$$

Example 4 Find $\displaystyle\sum_{n=1}^{n} \frac{1}{\sqrt{n+1}+\sqrt{n}}$ $\displaystyle\sum_{n=1}^{n} \frac{1}{\sqrt{n+1}+\sqrt{n}}$

Class 6

Example 1 Find S_n of $2(5) + 3(6) + 4(7) + - - -$

Example 2 Find $\displaystyle\sum_{n=1}^{n} \left(3n + 3^n + 3\right)$

Example 3 Find an expression for $\displaystyle\sum_{n \to 1}^{n} nx^n$ and hence find $\displaystyle\sum_{n \to 1}^{\infty} nx^n$ where $|x| < 1$.

Example 4 Find an expression for S_n of $5 + 55 + 555 + 5555 + - - -$

Example 5 Find S_n of $1 + (1+2) + (1+2+3) + - - - + (1+2+3+ - - - + n)$

Proofs Class

Proof 1 Prove the sum of the first n terms of an arithmetic series S_n is given by $S_n = \frac{n}{2}\{2a + (n-1)d\}$

Proof 2 To prove the sum of the first n terms of a geometric series S_n is given by

$$S_n = \frac{a(1-r^n)}{1-r}, \quad \text{where } |r| < 1$$

$$S_n = \frac{a(r^n - 1)}{r-1}, \quad \text{where } |r| > 1$$

Proof 3 To prove for an infinite geometric series $a + ar + ar^2 + ---$ is convergent if $|r| < 1$, then

$$S_\infty = \lim_{n\to\infty} S_n = \frac{a}{1-r}$$