The Line 4

This class contains area of a triangle, translations, central symmetry and axial symmetry.

Area of a triangle

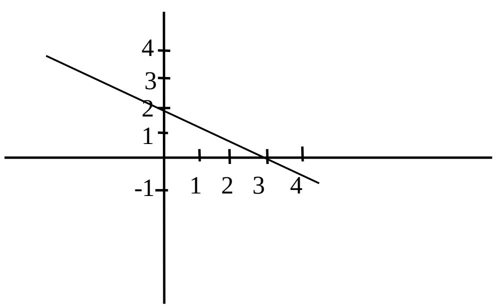
There are two different types of questions they can ask here but both will use the formula

$$\text{Area} = \frac{1}{2}\text{base} \times \text{perpendicular height}$$

Example 1 Find the area of the triangle formed by $2x + 3y = 6$ and the x and y axes.

Find where it cuts x - axis so sub in $y = 0$ to get $2x = 6$ so $x = 3$.
One point is (3,0)

Find where it cuts y - axis so sub in $x = 0$ to get $3y = 6$ so $y = 2$.
Second point is (0,2)



Base is the horizontal distance from to 0 to 3, which is 3.

Perpendicular distance is the distance from 0 to 2, which is 2.

$$\text{Area} = \frac{1}{2}(3)(2) = 3$$

Example 2 L is the line $2x+3y=8$ and M is the line $x-2y=-3$ find

(i) point p where L and M intersect
(ii) point q where L cuts the x - axis
(iii) point r where M cuts the x - axis
(iv) area of triangle pqr .

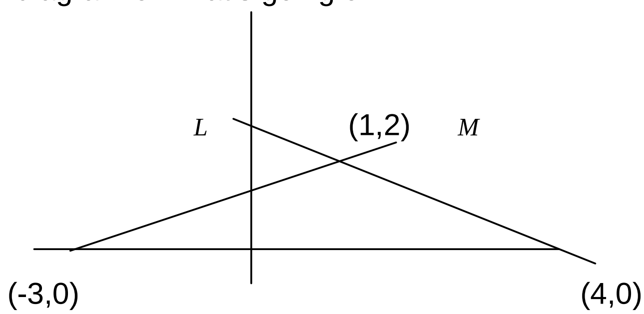
If you look back to example 5 of the last class we found the point of intersection by using simultaneous equations.

(i) Point of intersection is $p(1,2)$.

(ii) L cuts x - axis so sub in $y=0$ to get $2x=8$ so $x=4$. Point is $q(4,0)$

(iii) M cuts x - axis so sub in $y=0$ to get $x=-3$. Point is $r(-3,0)$

Draw a rough diagram of what's going on.



Base is the horizontal distance from to -3 to 4, which is 7.

Perpendicular distance is the distance from x - axis straight up to (1,2) which is 2.

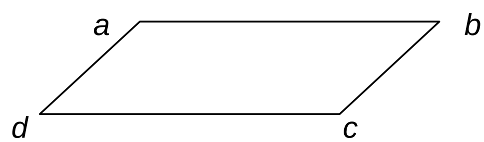
$$\text{Area} = \frac{1}{2}(7)(2) = 7$$

To move a point under a given Translation

Always attempt to make a rule out of the given translation

Example 3 a(-1,1), b(1,2), c(2,-1), and d (x,y) are four vertices in a parallelogram abcd. Find d.

Always make a rough diagram of the given question.



From the diagram we can see that d is in the left hand corner and that the movement from b to a is the same as going from c to d.

(1,2) ⟶ (-1,1) x down by 2, y down by 1

(2,-1) ⟶ (0,-2) x down by 2, y down by 1

Answer is d (0,-2)

Central symmetry

Definition – move through a point and go the same distance again the other side.

Example 4 Find image of (5,-2) under central symmetry in (3,-1)

x coordinates	5	down 2 to	3	down 2 to	1
y coordinates	-2	up 1 to	-1	up 1 to	0

Answer therefore is (1,0)

Axial Symmetry

Definition – move through a line at right angles and go the same distance again the other side.

Axial symmetry in the x-axis $=$ change the sign of y
Axial symmetry in the y-axis $=$ change the sign of x
Central symmetry in the origin $=$ change the sign of x and y.

The 3 rules above hold whether we are talking about a line or just a point.
Find the image of $2x-y=5$ under (i) S_x (ii) S_y (iii) S_o

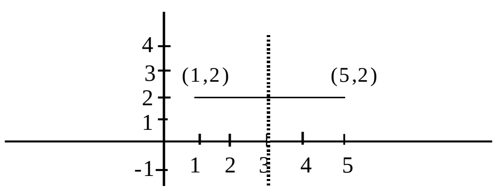
S_x axial symmetry in the x-axis $=$ change sign of y.
Answer $2x+y=5$

S_y axial symmetry in the y-axis $=$ change sign of x.
Answer $-2x-y=5$ or $2x+y=-5$

S_o central symmetry in the origin $=$ change sign of x and y.
Answer $-2x+y=5$ or $2x-y=-5$

Example 5 Find image of (1,2) under axial symmetry in $x=3$

Go straight across 2 spaces from 1 to 3 and then go 2 more across to 5.



Example 6 The equation of the line L is $x + 3y = 12$. The equation of the line M is $3x - y = k$.

(i) If $p(-2,8)$ is on M find the value of k.

(ii) Find the point q the point of intersection of L and M.

(iii) Prove $L \perp M$.

(iv) Find the point r, which is the image of p under axial symmetry in L.

(i) If $p(-2,8)$ is on M then sub in $x = -2$ and $y = 8$

$$3(-2) - 8 = k$$
$$-6 - 8 = k$$
$$k = -14$$

(ii) To find point of intersection use simultaneous equations.

$$x + 3y = 12 \quad \text{- - - } (i)$$
$$3x - y = -14 \text{ - - - } (ii)$$

$$x + 3y = 12$$
$$\underline{9x - 3y = -42}$$
$$10x \quad = \quad -30$$
$$x \quad = \quad -3$$

Sub $x = -3$ into equation (i) $-3 + 3y = 12$

$$3y = 15$$
$$y = 5$$

q is the point (–3,5)

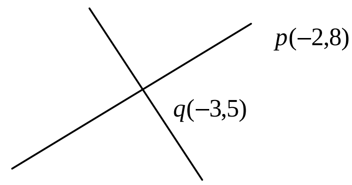
(iii) Slope of L is $m_1 = -\frac{1}{3}$

Slope of M is $m_2 = -\frac{3}{-1} = \frac{3}{1}$

$L \perp M$ since $m_1 m_2 = \left(-\frac{1}{3}\right)\left(\frac{3}{1}\right) = -1$

(iv) Draw a diagram to see what is going on.

We know that for axial symmetry we must go through a line at right angles and go the same again the other side. The question has already proven that the lines are at right angles to each other and has also found the point of intersection so we can change the axial symmetry into a central symmetry by moving $p(-2,8)$ through $q(-3,5)$ and onto r.



x coordinates	-2	down 1 to	-3	down 1 to	-4
y coordinates	8	down 3 to	5	down 3 to	2

Answer therefore is $r(-4,2)$