

Probability 3

This class contains probability.

Basic Probability

The probability of a certain event, E, taking place written as P (E) is given by

$$P(E) = \frac{\text{number of times E occurs}}{\text{number of times E could occur}}$$

$$0 \leq P(E) \leq 1$$

$P(E) = 0$ means E cannot happen.

$P(E) = 1$ means E is a certainty.

$P(E \text{ does not happen}) = 1 - P(E \text{ does happen})$

Sample space is the set of all possible outcomes of an experiment.

The Addition Rule / Or Rule

(a) If E and F are two events mutually exclusive (cannot occur together) then

$$P(E \text{ or } F) = P(E) + P(F)$$

(b) If E and F are two events, which may occur at the same time then

$$P(E \text{ or } F) = P(E) + P(F) - P(E \text{ with } F)$$

The multiplication Rule / And Rule

$$P(E \text{ and } F) = P(E) \times P(F).$$

Example 1 What is the probability of drawing a white ball from a bag with 4 white and 7 black balls.

$$P(\text{ white ball}) = \frac{4}{11}$$

Mutually Exclusive Events

Mutually Exclusive Events are events that cannot occur at the same time. There is no overlap.

Example 2 A coin and a dice are thrown at the same time. Find the probability of getting a head on the coin or a 6 on the dice.

$$P(H) = \frac{1}{2}$$

$$P(6) = \frac{1}{6}$$

$$P(H \text{ or } 6) = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

The probability of a King or a heart does have something in common i.e the King of hearts.

Example 3 A card is drawn from a pack at random what is the probability that the card drawn is

- (a) a Queen
- (b) a Spade
- (c) a Queen or a Spade.

There are 52 cards in a pack (sample space is 52)

$$P(\text{Queen}) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{Spade}) = \frac{13}{52} = \frac{1}{4}$$

$$\begin{aligned} P(\text{Queen or a Spade}) &= P(\text{Queen}) + P(\text{Spade}) - P(\text{Queen of Spade}) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{15}{52} = \frac{15}{52} \end{aligned}$$

Independent events

Example 4 A coin and a dice are thrown at the same time. Find the probability of getting a head on the coin and a 6 on the dice.

$$P(H) = \frac{1}{2}$$

$$P(6) = \frac{1}{6}$$

$$P(H \text{ and } 6) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

Example 5 In a bag containing 11 marbles, seven are red and four are black. Find the probability that when a marble is drawn at random and replaced and a second and third marble is drawn and replaced that

- (i) all three where the same colour.
- (ii) One was red.
- (iii) At least two marbles where red.

Total of 11. 7 are red and 4 are black.

(i) $P(\text{all same colour}) = P(\text{all red}) + P(\text{all black})$

$$P(\text{all red}) = \frac{7}{11} \times \frac{7}{11} \times \frac{7}{11} = \frac{343}{1331}$$

$$P(\text{all black}) = \frac{4}{11} \times \frac{4}{11} \times \frac{4}{11} = \frac{64}{1331}$$

$$P(\text{all same colour}) = \frac{343}{1331} + \frac{64}{1331} = \frac{407}{1331} = \frac{37}{121}$$

(ii) $P(\text{one red}) = P(R) \times P(B) \times P(B)$ this answer must be multiplied by 3 as the red can be arranged in 3 ways.

$$P(\text{one red}) = 3 \times \frac{7}{11} \times \frac{4}{11} \times \frac{4}{11} = \frac{336}{1331}$$

(iii) $P(\text{at least two red}) = 1 - (P(\text{one red}) + P(\text{none red}))$

$$P(\text{none red}) = P(\text{all black})$$

$$P(\text{at least two red}) = 1 - \left(\frac{336}{1331} + \frac{64}{1331} \right)$$

$$= \frac{931}{1331}$$

Dependent events

Example 6 A bag contains 4 blue and 5 black disks one disk is taken out and not replaced and a second is taken out. Find the probability that

- (i) the second one was blue,
- (ii) both were blue,
- (iii) at least one was blue.

(i) The second blue. Must write out the possible ways this can happen.

$$P(\text{second one blue}) = P(\text{blue and blue}) \text{ or } P(\text{black and blue})$$

$$P(\text{blue and blue}) = \frac{4}{9} \times \frac{3}{8} = \frac{3}{18} = \frac{1}{6}$$

$$P(\text{black and blue}) = \frac{5}{9} \times \frac{4}{8} = \frac{5}{18}$$

$$P(\text{second one blue}) = \frac{3}{18} + \frac{5}{18} = \frac{8}{18} = \frac{4}{9}$$

(ii) Both blue.

$$P(\text{both blue}) = P(\text{blue}) \text{ and } P(\text{blue}) = \frac{4}{9} \times \frac{3}{8} = \frac{3}{18} = \frac{1}{6}$$

(iii) At least one blue.

$$P(\text{at least one blue}) = 1 - P(\text{both black})$$

$$P(\text{both black}) = P(\text{black}) \text{ and } P(\text{black}) = \frac{5}{9} \times \frac{4}{8} = \frac{5}{18}$$

$$P(\text{at least one blue}) = 1 - \frac{5}{18} = \frac{13}{18}$$

Example 7 If there are 6 gold and x silver coins in a bag and one is taken out and not replaced and a second is taken out if the probability that both coins are silver is $\frac{4}{13}$ find the value of x .

$$P(\text{S and S}) = \frac{x}{6+x} \cdot \frac{x-1}{5+x} = \frac{4}{13}$$

$$13x(x-1) = 4(6+x)(5+x)$$

$$13x^2 - 13x = 4(30 + 11x + x^2)$$

$$13x^2 - 13x = 120 + 44x + 4x^2$$

$$9x^2 - 57x - 120 = 0$$

$$3x^2 - 19x - 40 = 0$$

$$(3x+5)(x-8) = 0$$

$$x = -\frac{5}{3} \text{ or } x = 8$$

Answer is $x = 8$ since answer must be whole and positive.

Birthday question

Example 8 Three people were asked "on what day of the week were you born?".

Find the probability that

- (i) all were born on Monday,
- (ii) all born on the same day,
- (iii) all born on different days,
- (iv) at least two were born on the same day of the week.

$$(i) P(\text{Mon and Mon and Mon}) = \frac{1}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{243}$$

$$(ii) P(\text{born and same and same}) = \frac{7}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$$

$$(iii) P(\text{born and different day and different day}) = \frac{7}{7} \times \frac{6}{7} \times \frac{5}{7} = \frac{30}{49}$$

$$(iv) P(\text{at least 2 the same}) = 1 - P(\text{all different}) = 1 - \frac{30}{49} = \frac{19}{49}$$

Note In parts (ii) and (iii) the first person is born before we figure the rest out.

Question 6(c) of 2009

(b) In a class of 24 students, there are 14 boys and 10 girls.

In a particular week (Monday to Sunday inclusive), three students celebrate their birthday. Assume that the birthdays are equally likely to fall on any day of the week and that the birthdays are independent of each other.

What is the probability that these three students

- (i) are three boys or three girls
- (ii) have birthdays falling on different days of the week or the same day of the week other than Monday?

14 boys and 10 girls.

(i) $P(B)$ and $P(B)$ and $P(B)$ or $P(G)$ and $P(G)$ and $P(G)$

$$\frac{14}{24} \times \frac{13}{23} \times \frac{12}{22} + \frac{10}{24} \times \frac{9}{23} \times \frac{8}{22}$$

$$\frac{91}{506} + \frac{15}{253} = \frac{11}{46}$$

(ii) Birthdays falling on different days of the week.

$P(\text{born})$ and $P(\text{born same day})$ and $P(\text{born same day})$

$$1 \times \frac{6}{7} \times \frac{5}{7} = \frac{30}{49}$$

The same day of the week other than Monday.

$$\frac{6}{7} \times \frac{1}{7} \times \frac{1}{7} = \frac{6}{343}$$

$$\text{Answer } \frac{30}{49} + \frac{6}{343} = \frac{216}{343}$$

Sample Space

Example 9 If two unbiased dice are tossed find the probability that

- (i) both dice show odd numbers,
- (ii) the sum of the results is 9 or greater.

This is an example of sample space where we write out all the possible answers.

11	21	31	41	51	61
12	22	32	42	52	62
13	23	33	43	53	63
14	24	34	44	54	64
15	25	35	45	55	65
16	26	36	46	56	66

- (i) There are 36 answers and we have to count up the number of answers which are both odd

11 31 51 13 33 53 15 35 55

$$\text{Answer } \frac{9}{36} = \frac{1}{4}$$

- (ii) Find the results that add to 9 or more.

36 45 46 54 55 56 63 64 65 66

$$\text{Answer } \frac{10}{36} = \frac{5}{18}$$

Example 10 On an unbiased die, the numbers 1, 3 and 4 are coloured red and the numbers 2, 5 and 6 are coloured black.

- (i) The die is thrown once. Find the probability of getting an even number or a red number.
- (ii) The die is thrown three times with the following outcome:

The second throw shows a red number and the sum of the numbers on the first and second throw is equal to the number on the third throw.

Find the probability of this outcome.

- (i) 3 even numbers and two different numbers that are red.

$$P(\text{red or even}) = \frac{5}{6}$$

- (ii) Second throw is red so is 1, 3 or 4

The sum of the first two add to the third so they cannot be more than 6. The only real way to figure this question out is by sample space.

1 st	2 nd	3 rd
1	1	2
2	1	3
3	1	4
4	1	5
5	1	6
1	3	4
2	3	5
3	3	6
1	4	5
2	4	6

Total of 10 results.

How many possible results are there?

There are 6 for the first dice and 6 for the second and third so total number of answers is $6 \times 6 \times 6 = 216$.

Answer to our question is $\frac{10}{216} = \frac{5}{108}$

Permutations and Probability

Example 11 Find the probability that when a, b, c, d, e are arranged that the vowels are together?

$$P(\text{together}) = \frac{\text{together}}{\text{total}}$$

$$\text{Total} = 5! = 120$$

$$\text{Together} = 2(4!) = 2(24) = 48$$

$$P(\text{together}) = \frac{48}{120} = \frac{2}{5}$$

Example 12 A classroom contains 15 desks which are arranged in rows. The front row contains 3 desks. 15 students are seated at random in the classroom, 8 of whom are boys and 7 of whom are girls. Each desk seats only one student.

What is the probability that

- (i) three girls occupy the front row of desks?
- (ii) there are more boys than girls seated in the front row?
- (iii) there are two girls and one boy seated in the front row with the two girls seated next to each other?

8 boys and 7 girls

- (i) P(girl and girl and girl)

$$\frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} = \frac{1}{13}$$

- (ii) P(boy and boy and girl) or P(boy and boy and boy)

P(boy and boy and girl)

$$\frac{8}{15} \times \frac{7}{14} \times \frac{7}{13} \times 3 = \frac{28}{65}$$

P(boy and boy and boy)

$$\frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} = \frac{8}{65}$$

$$\text{Answer } \frac{28}{65} + \frac{8}{65} = \frac{36}{65}$$

(ii) P(girl and girl and boy)

$$\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \times 2 = \frac{16}{65}$$